Digital Signal Processing
Concepts and Theory

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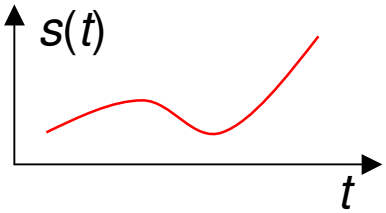
What is a DSP?

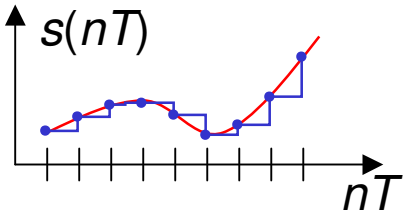
- Digital
  - ➔ Operating by the use of discrete signals to represent data in the form of numbers
- Signal
  - ➔ A variable parameter by which information is conveyed through an electronic circuit
- Processing
  - ➔ To perform operations on data according to programmed instructions
- Digital Signal Processing
  - ➔ Changing or analysing information which is measured as discrete sequences of numbers

Signals

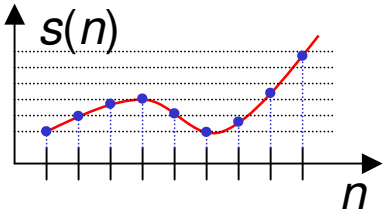
- Continuous-time (analog) signals



- Discrete-time signals



- Digital signals

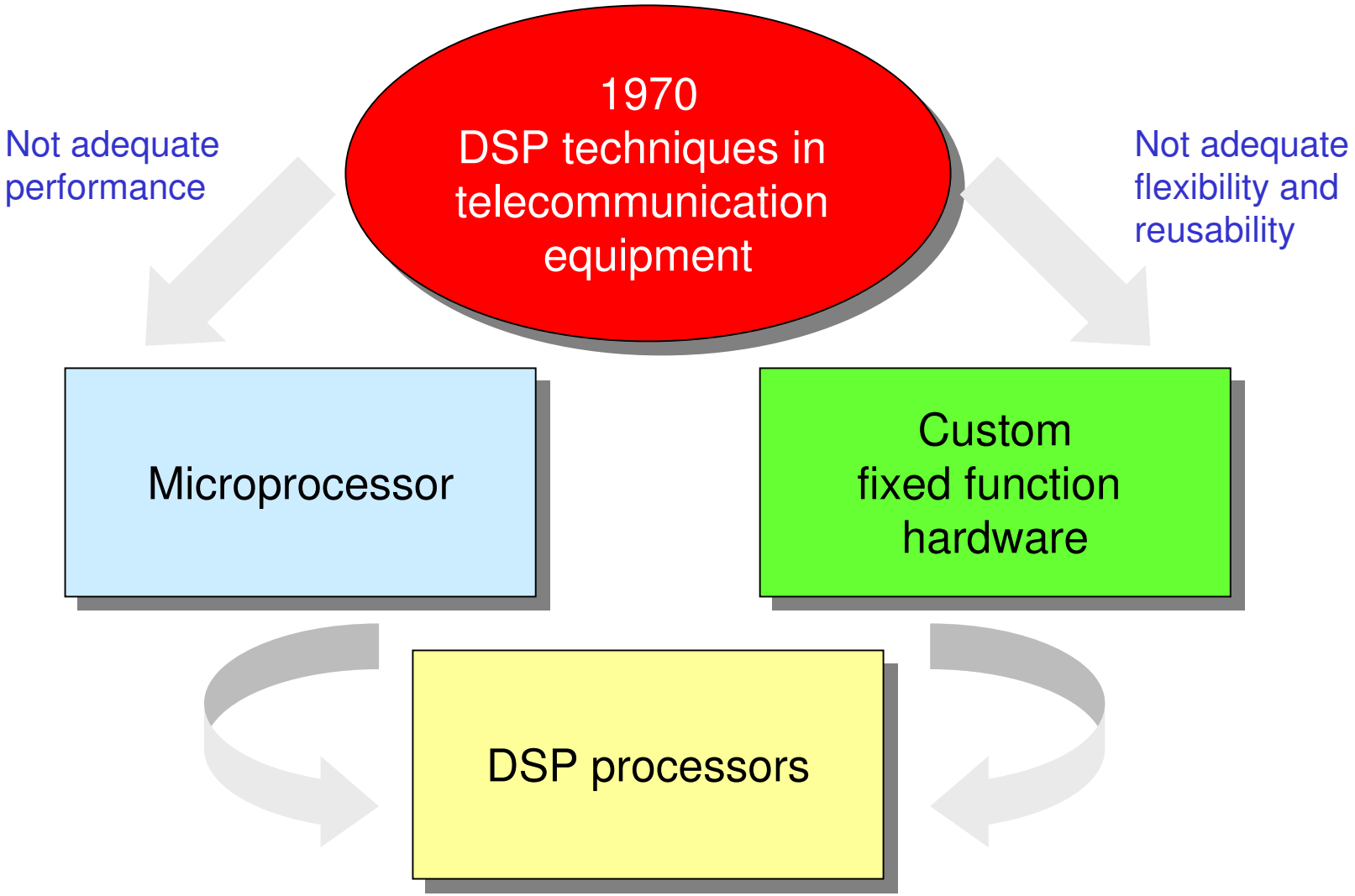


Signal	Time	Amplitude
Analog	Continue	Continue
Discrete	Discrete	Continue
Digital	Discrete	Discrete

Main Characteristics

- Compared to other embedded computing applications, DSP applications are differentiated by the following
  - ➔ Computationally demanding
    - ✓ Iterative numeric algorithms
  - ➔ Sensitivity to small numeric errors (audible noise)
  - ➔ Stringent real-time requirements
  - ➔ Streaming data
  - ➔ High data bandwidth
  - ➔ Predictable (though often eccentric) memory access pattern
  - ➔ Predictable program flow (nested loops)

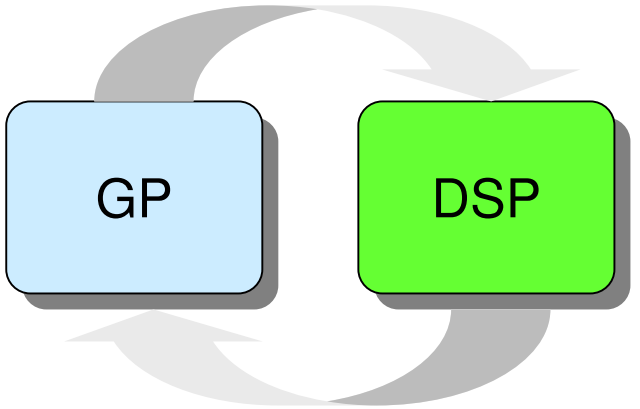
DSP Processors



DSP vs. General Purpose

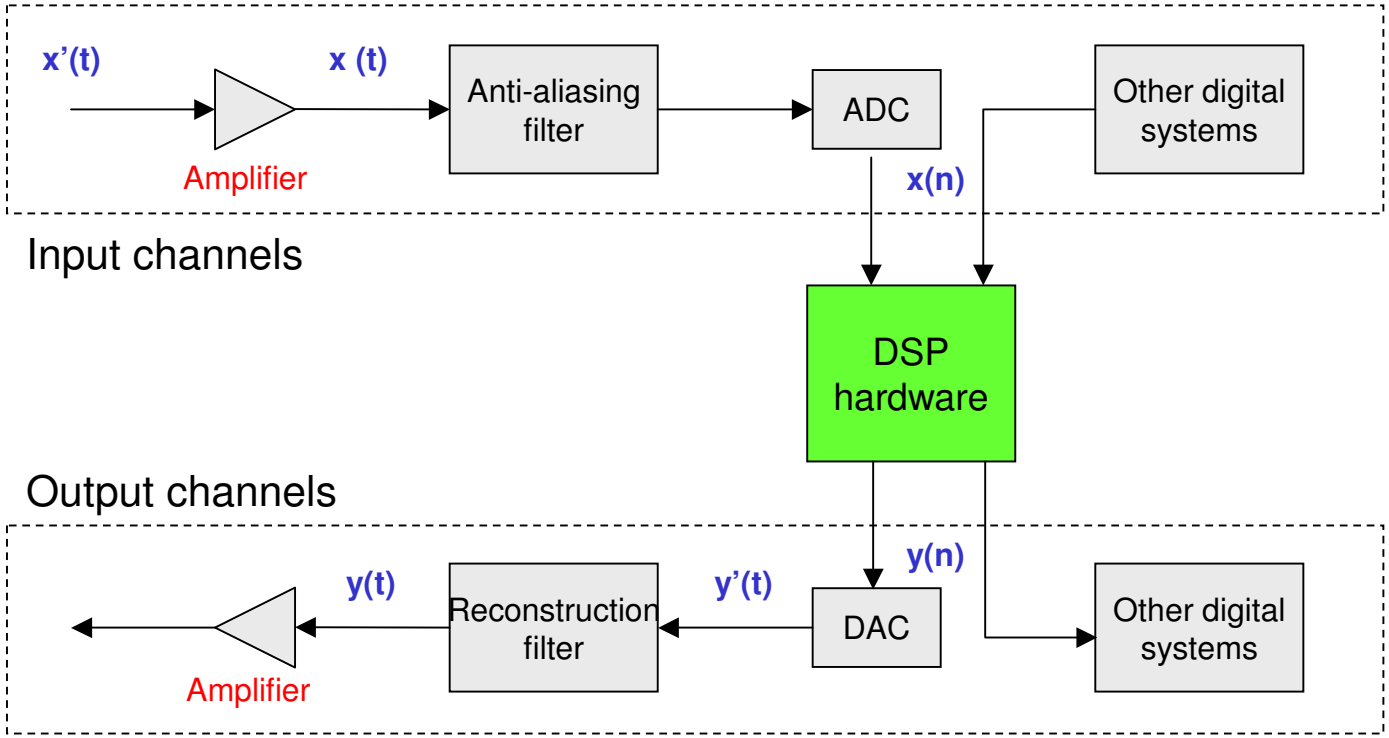
- DSPs adpot a range of specialized features
  - Single-cycle multiplier
  - Multiply-accumulate operations
  - Saturation arithmetic
  - Separate program and data memories
  - Dedicated, specilized addressing hw
  - Complex, specialized instruction sets

VLIW, Superscalar, SIMD, multiprocessing, ...

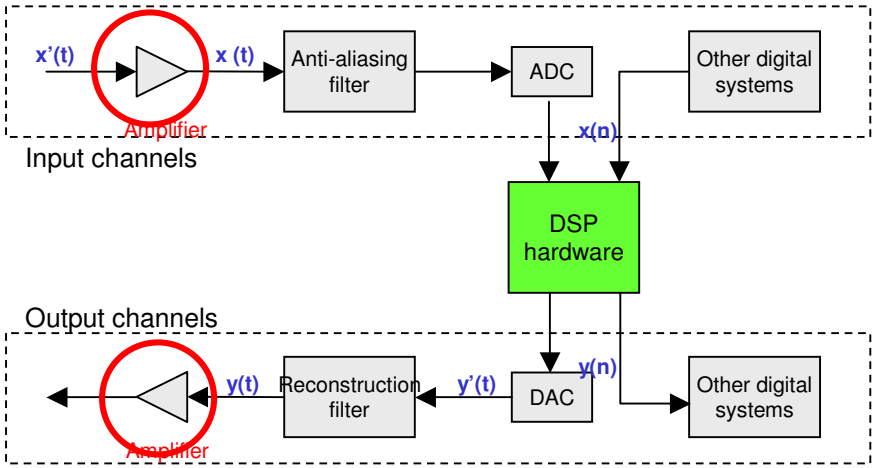


- Today, virtually very commercial 32-bit microprocessor architecture (from ARM to 80x86) has been subject to some kind of DSP-oriented enhancement

Basic Elements of DSP Systems

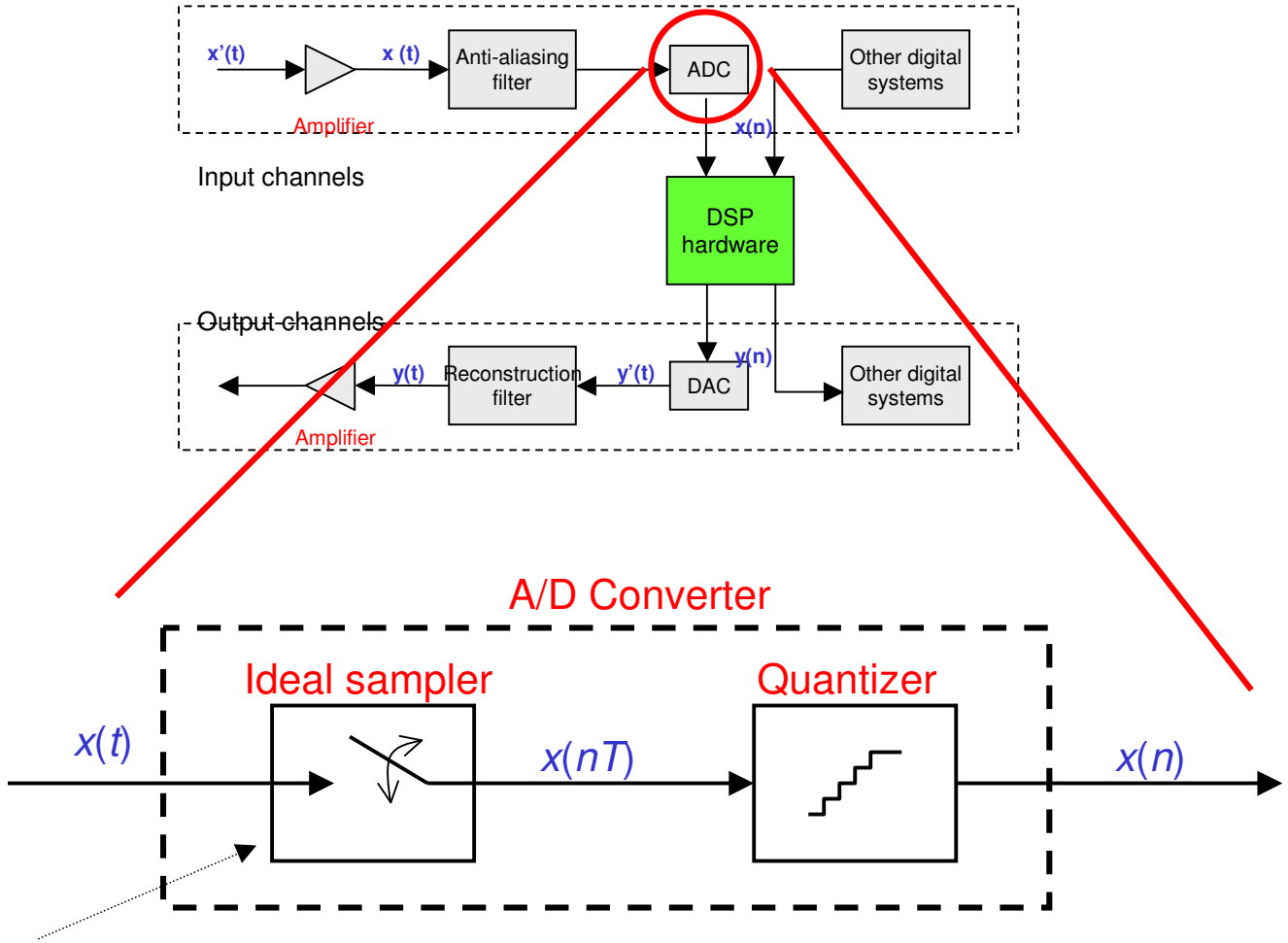


Amplifier



- $x(t)=gx'(t)$
- The gain g is determined such that $x(t)$ has a dynamic range that matches the ADC
- Very difficult in practice
  - ➔ $x'(t)$ may be unknown and changing with time
    - ✓ Especially for signals with larger dynamic range (e.g., speech)

ADC



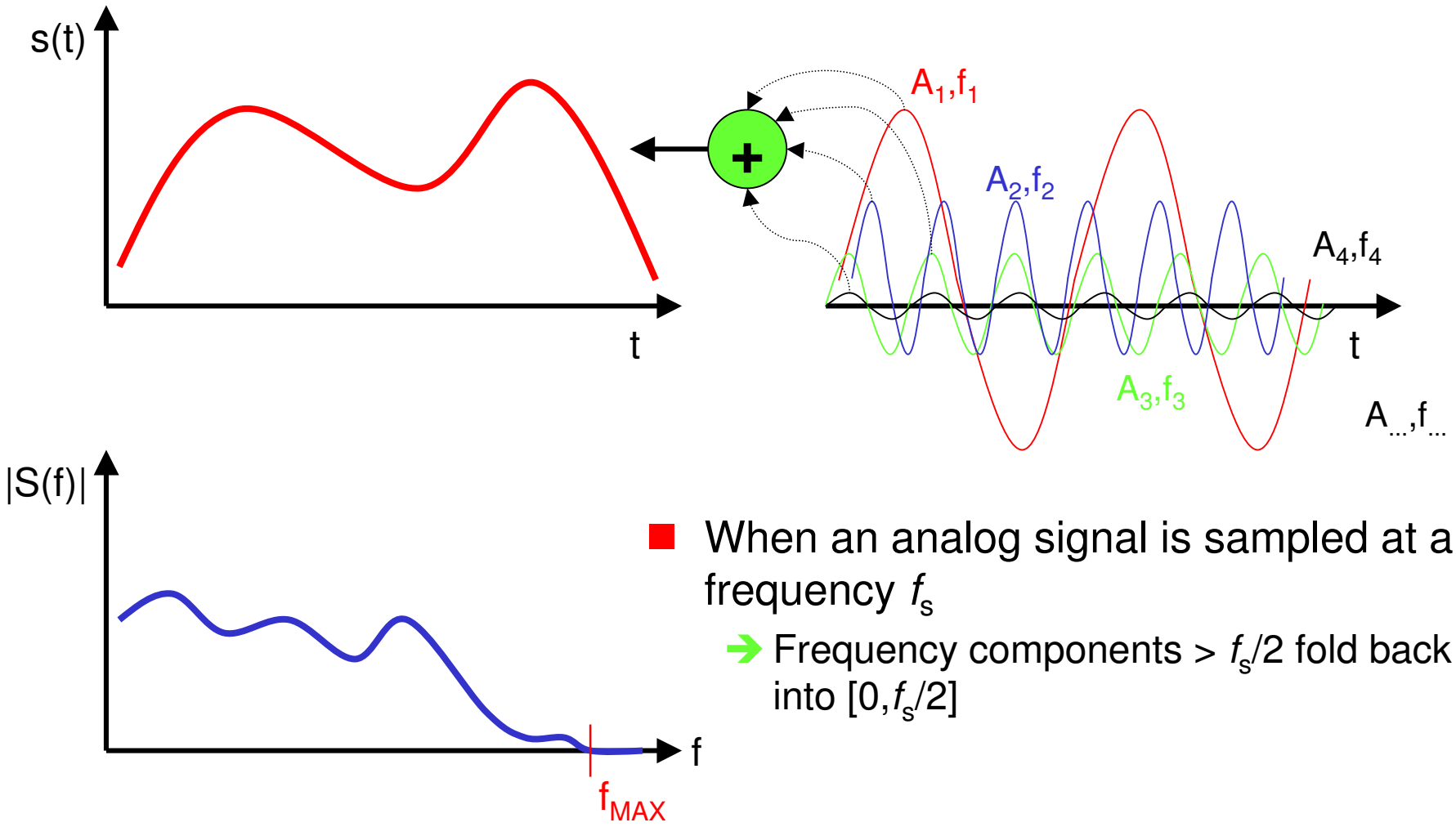
Sample-and-hold circuit

Sampling

- In order to represent an analog signal $x(t)$ by $x(nT)$ accurately, two conditions must be met
  - ➔ $x(t)$ must be bandlimited by f_M
  - ➔ The sampling frequency, f_s, must be $>2f_M$

- Shannon's theorem
  - ➔ When the sampling frequency is greater than twice the highest frequency component contained in $x(t)$, it can be perfectly reconstructed from $x(nT)$

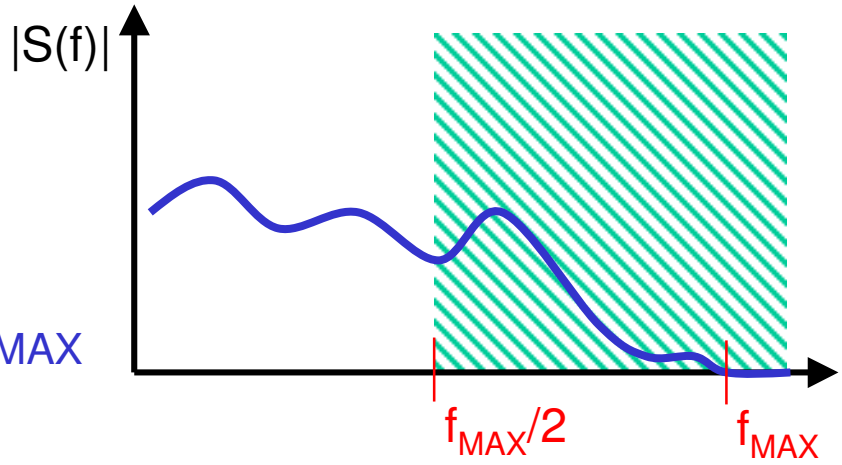
Folding Effect (1/2)

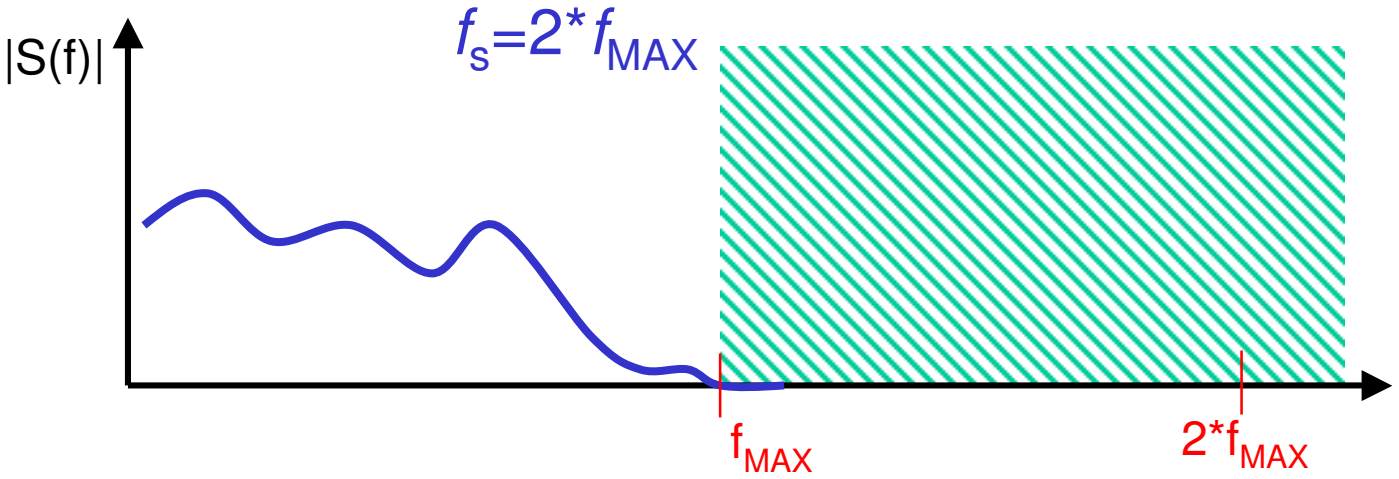


- When an analog signal is sampled at a frequency f_s
  - ➔ Frequency components > $f_s/2$ fold back into $[0, f_s/2]$

Folding Effect (2/2)

- When an analog signal is sampled at a frequency f_s
  - ➔ Frequency components > $f_s/2$ fold back into $[0, f_s/2]$

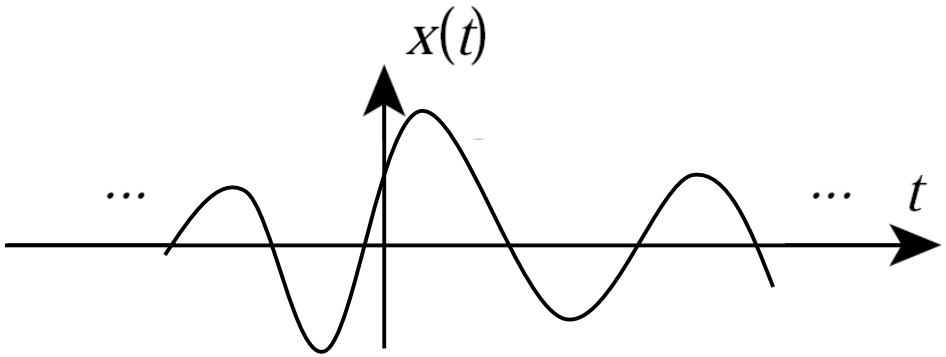




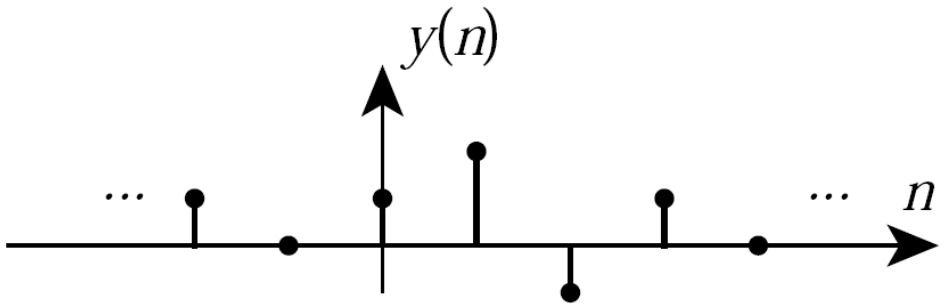
Sampling and Reconstruction

- Formal explaination of
  - Aliasing
  - Reconstruction

Sampling



T: sampling interval (sec)
F_s: sampling frequency (Hz)
$\omega_s = 2\pi/T$ (rad/s)



$$y(n) = x(nT) \quad \forall n \in Ints$$

Aliasing

$$x(t) = \cos(2\pi f t) \qquad u(t) = \cos(2\pi(f + Nf_s)t)$$

$$y = Sampler_T(x) \Rightarrow y(n) = \cos(2\pi f n T)$$

$$w = Sampler_T(u) \Rightarrow w(n) = \cos(2\pi(f + Nf_s)nT) =$$

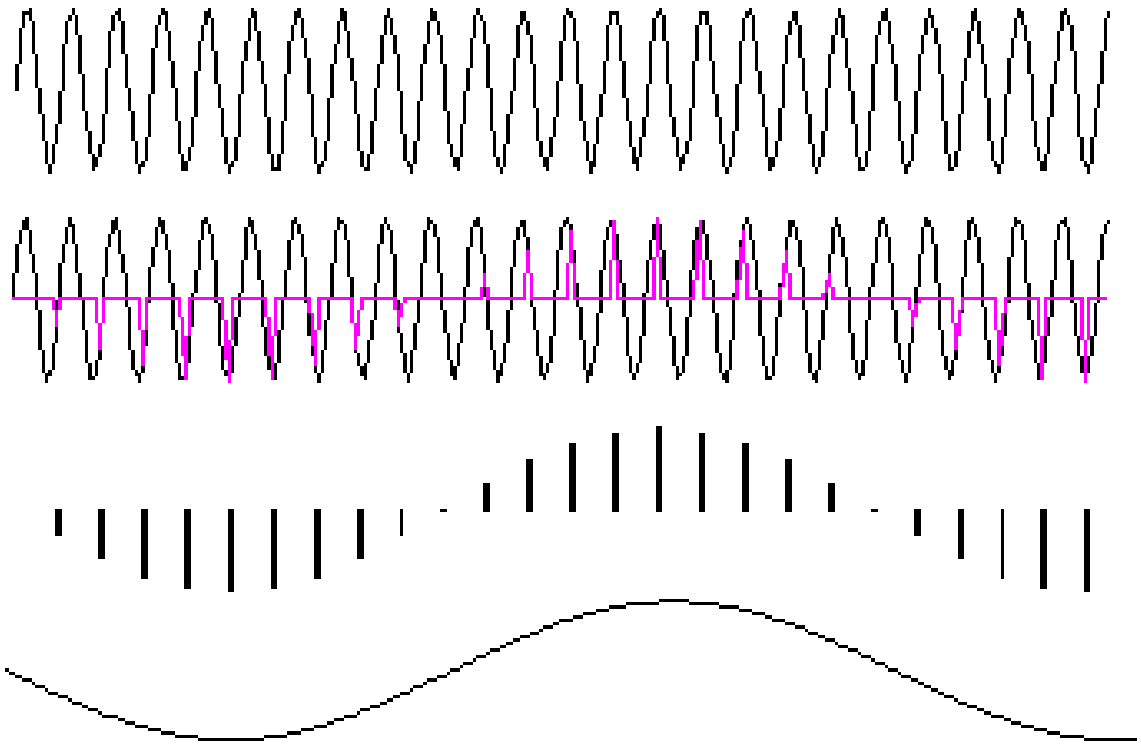
$$= \cos(2\pi f n T + 2\pi N n) = \cos(2\pi f n T) =$$

$$= y(n)$$

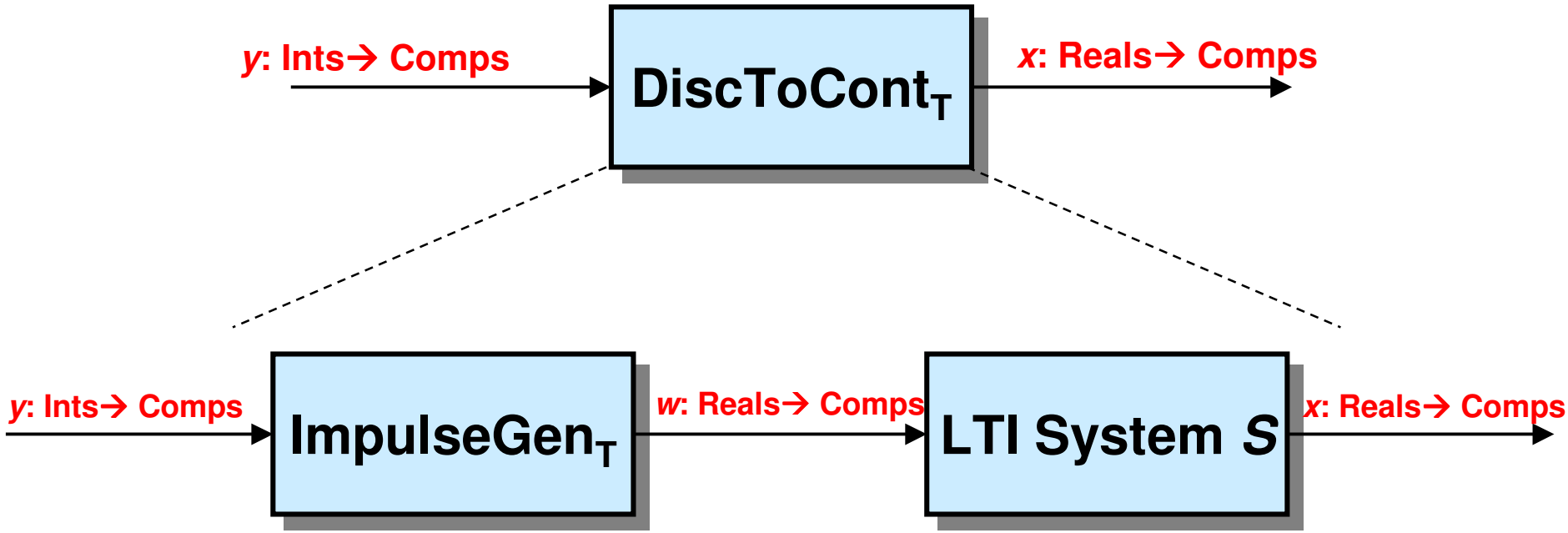
- Even though ***u≠x***, ***$Sampler_T(u) = Sampler_T(x)$***
  - ➔ After sampling, ***x*** and ***u*** are indistinguishable

Aliasing

- Some higher frequencies can be incorrectly interpreted
  - ➔ Aliasing problem: One frequency looks like another

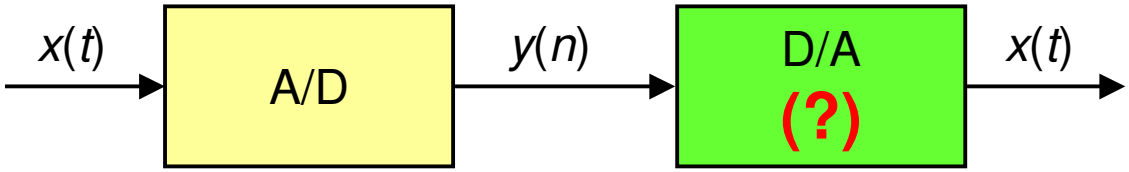


Reconstruction



$$x = S(ImpulseGen_T(y))$$

Three-step Reconstruction Proof



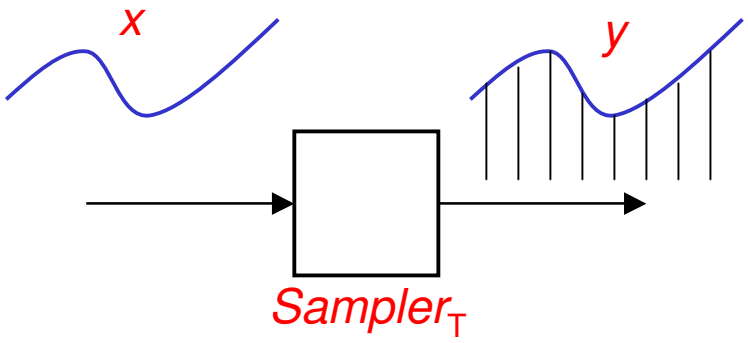
- Let $x(t)$ be a continuous-time signal with Fourier transform X
- Let $X(\omega)$ be zero outside the range $-\pi/T<\omega<\pi/T$

1st Step

- Sampling *x* with sampling interval T

$$y = Sampler_T(x)$$

$$y(n) = x(nT)$$



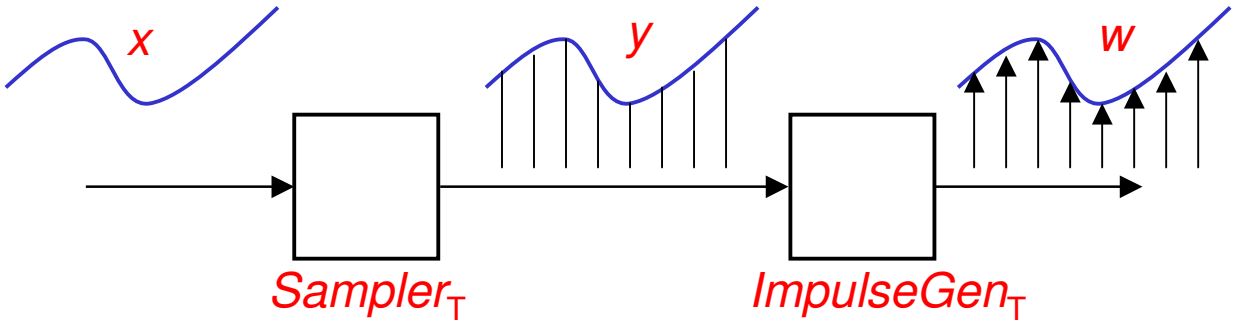
- It can be shown that

$$Y(\omega) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X\left(\frac{\omega - 2\pi k}{T}\right)$$

2nd Step

- Let *w* be the signal produced by the

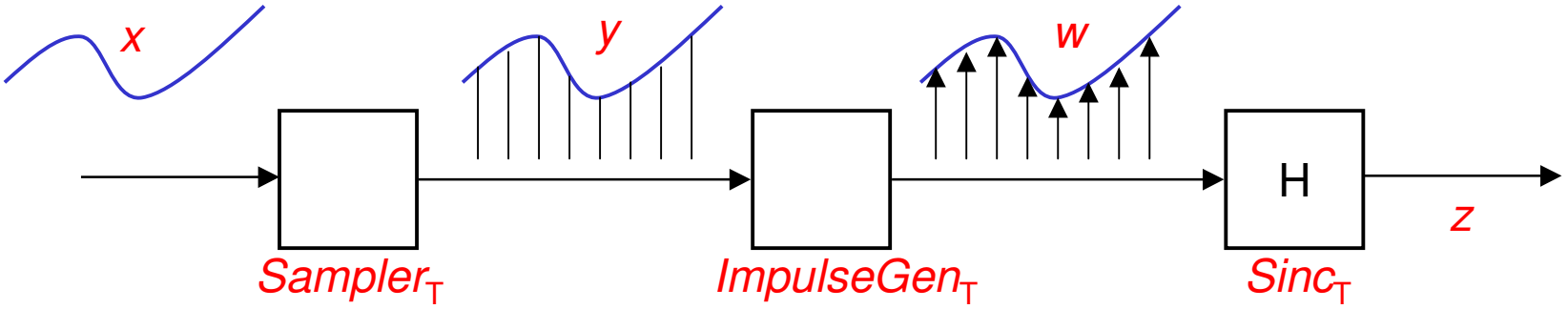
$$w(t) = \sum_{n=-\infty}^{+\infty} y(n)\delta(t - nT)$$



- It can be shown that

$$W(\omega) = Y(\omega T)$$

2nd Step (cont.)



- Let *z* be the output of the *Sinc*$_T$

$$Z(\omega) = W(\omega)H(\omega) = Y(\omega T)H(\omega)$$

Frequency response of the *reconstruction filter Sinc*$_T$

2nd Step (cont.)

- The frequency response of the reconstruction filter is

$$H(\omega) = \begin{cases} T & -\pi/T < \omega < \pi/T \\ 0 & \text{otherwise} \end{cases}$$

- Substituting for *H* and *Y*, we get

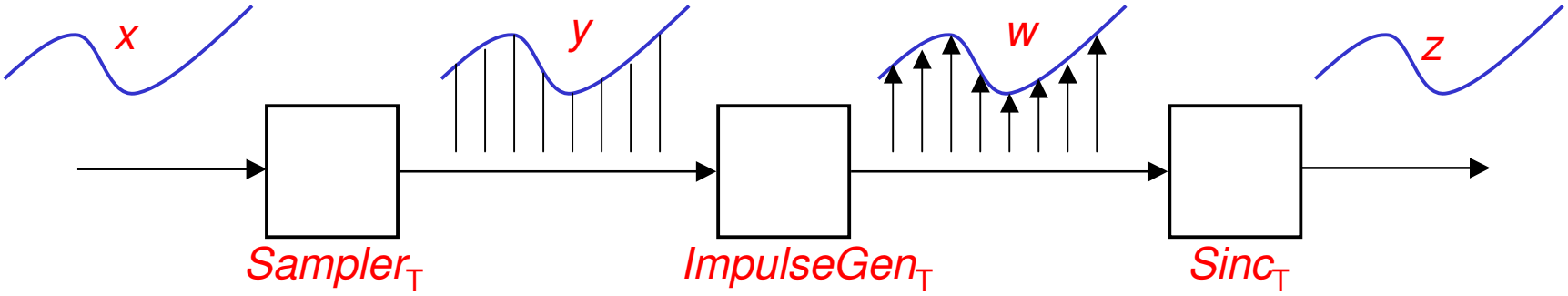
$$Z(\omega) = \begin{cases} \sum_{k=-\infty}^{\infty} X(\omega - 2\pi k/T) & -\pi/T < \omega < \pi/T \\ 0 & \text{otherwise} \end{cases}$$

3rd Step

- As $X(\omega)$ is zero for $|\omega|$ larger than the Nyquist frequency π/T, then

$$Z(\omega) = X(\omega) \qquad \forall \omega \in \Re$$

- That is, *z* is identical to *x*

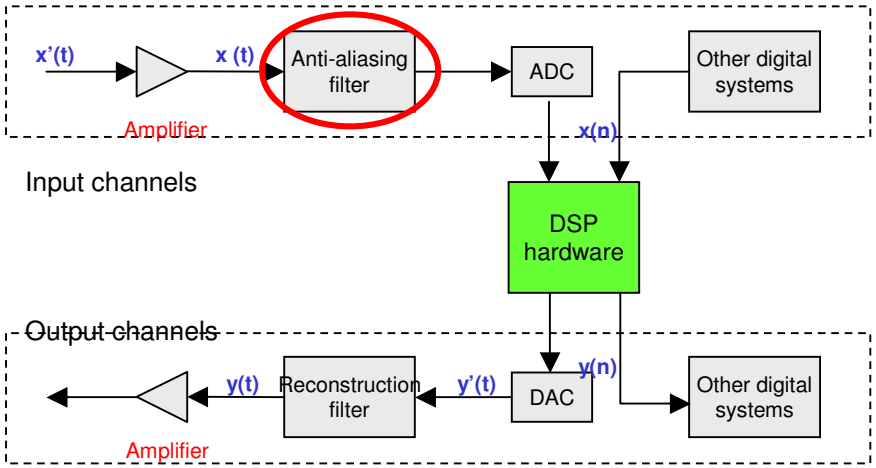


$$x(t) = z(t) = F^{-1}[Z(\omega)] = \sum_{k=-\infty}^{+\infty} x(nT) \cdot \mathrm{sinc}\left(\frac{t - nT}{T}\right)$$

Practical Applications (1/2)

- In general $x(t)$ is not bandlimited
- Significant energies outside the highest frequency of interest
- Voice communication systems use 8 kHz sampling rate
  - ➔ Maximum frequency component in a speech signal is much higher than 4kHz
  - ➔ Out-of-band components at the input of ADC can become in-band because the folding effect

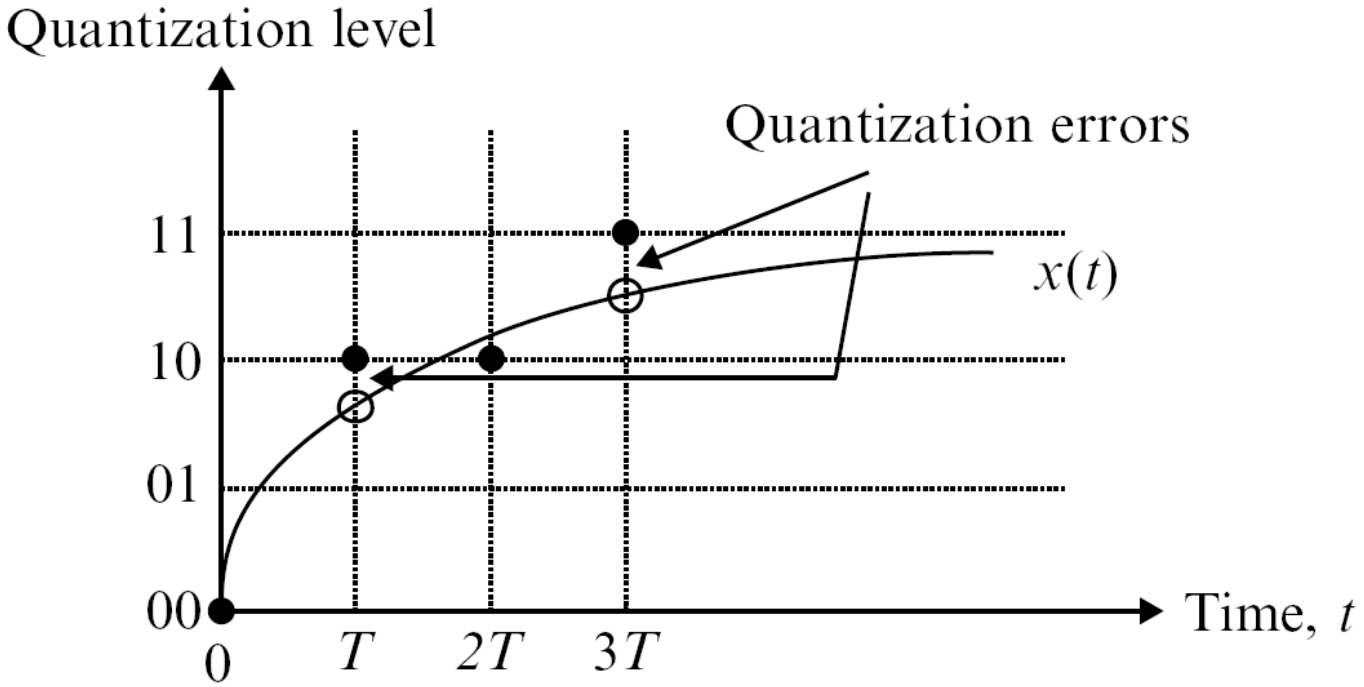
Practical Applications (2/2)



- Solution
  - ➔ Filter $x(t)$ with a lowpass filter (*anti-aliasing filter*)
  - ➔ In many practical system a bandpass filter is used
    - ✓ In order to prevent undesidered DC offset

Quantizing and Encoding

- $x(nT)$ assume infinite wordlength
- $x(n)$ assume finite wordlength



Digital samples using 2-bit quantizer.

Signal to Noise Ratio (SNR)

- The *Signal to noise ratio* (SNR) is the ratio of a signal power to the noise power corrupting the signal
  - ➔ It compares the level of a desired signal to the level of background noise
  - ➔ The higher the ratio, the less obtrusive the background noise is

SNR

- Signal-to-quantization Noise Ratio is approximated by

$$\text{SNR} \approx 6B \text{ dB}$$

  - ➔ Where B is the wordlength
- This is a theoretical maximum
  - ➔ Simple guideline for determing the wordlenght of the quantizer for a given application

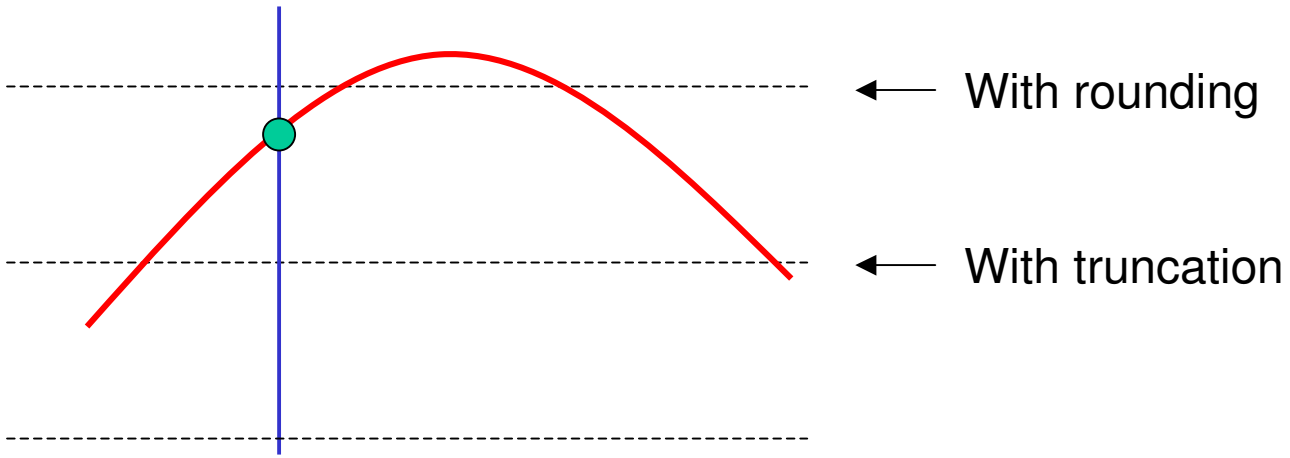
Input Quantization Noise

- Let us assume that $x(n)$ is scaled such that $-1 \leq x(n) < 1$
- If the quantizer employs B bits
  - ➔ The number of quantization levels for representing $x(nT)$ is 2^B
  - ➔ The spacing between two quantization levels is

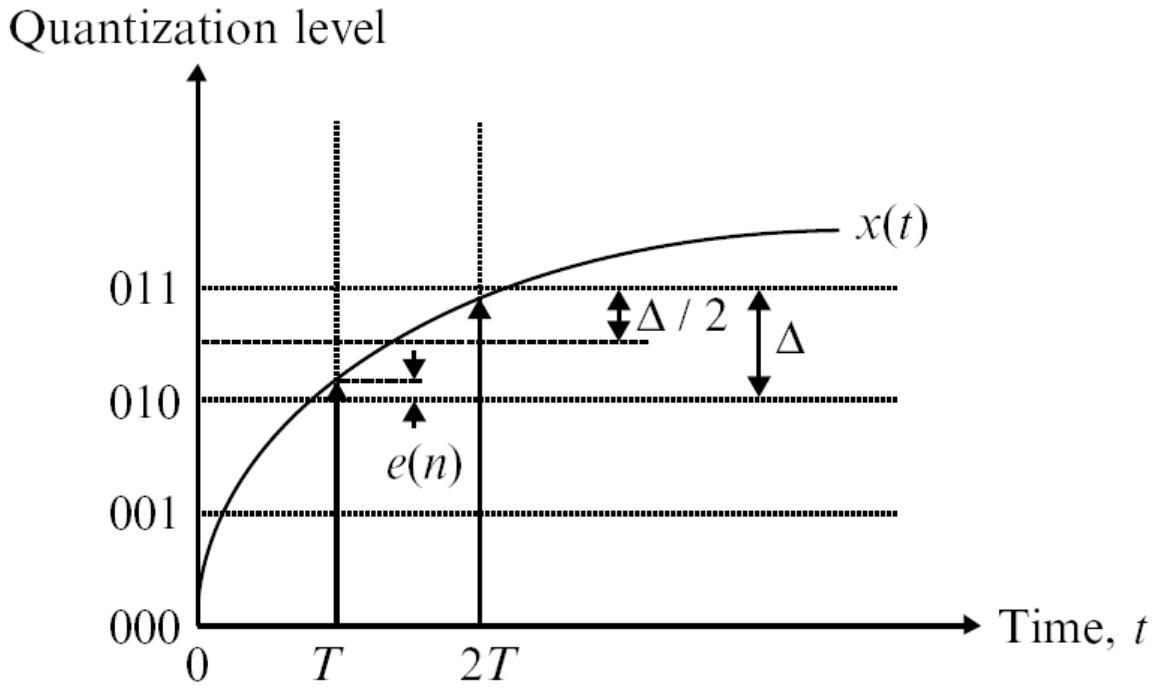
$$\Delta = \frac{\text{full scale range}}{\text{number of quantization levels}} = \frac{2}{2^B} = 2^{-B+1} = 2^{-M}$$

Common Methods of Quantization

- Rounding
  - ➔ The signal value is approximated using the nearest quantization level
- Truncation
  - ➔ The signal value is approximated using the highest quantization level that is not greater than the signal itself
    - ✓ Produces bias effects

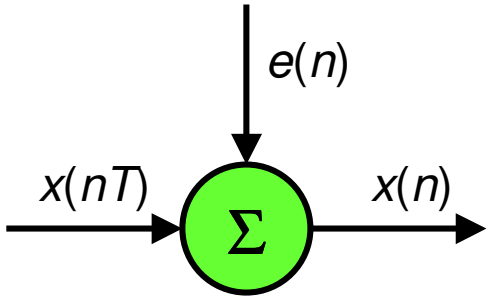


Quantization Error



$$e(n) = x(n) - x(nT)$$

$$|e(n)| \leq \frac{\Delta}{2}$$



Quantization Error (mean)

- For an arbitrary signal with fine quantization (*B* is large), *e*(*n*) may be assumed to be uncorrelated with *x*(*n*)
  - ➔ Can be assumed to be random noise that is uniformly distribuited in [-Δ/2, Δ/2]

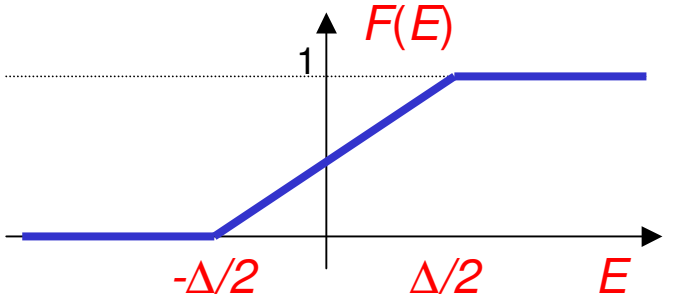
$$E[e(n)] = \frac{-\Delta/2 + \Delta/2}{2} = 0$$

The quantization noise e(n) has zero mean.

Quantization Error (variance)

$$\sigma_e^2 = E\left[(e - m_e)^2\right] = E\left[e^2\right] = \int_{-\infty}^{+\infty} E^2 f(E)\,dE =$$

$$= \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} E^2\,dE = \frac{\Delta^2}{12} = \frac{2^{-2B}}{3}$$



The larger the wordlength, the smaller the input quantization error.

SQNR

- If the quantization error is regarded as noise

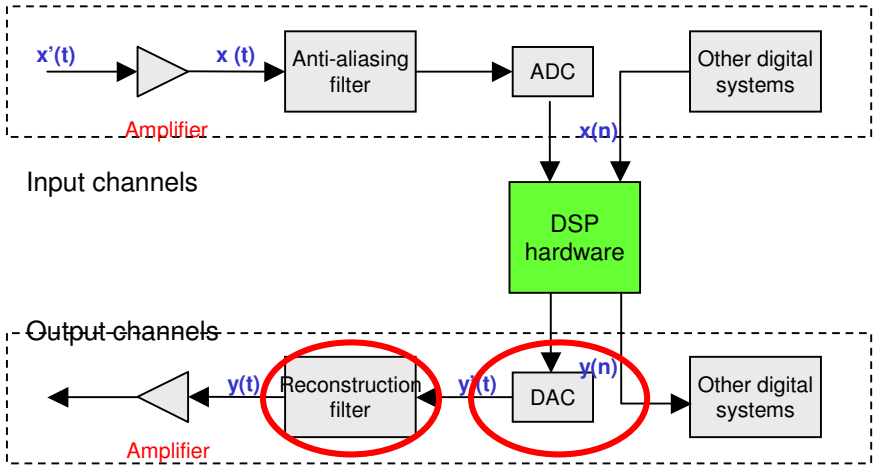
$$SQNR = \frac{\sigma_x^2}{\sigma_e^2} = 3 \cdot 2^{2B} \sigma_x^2$$

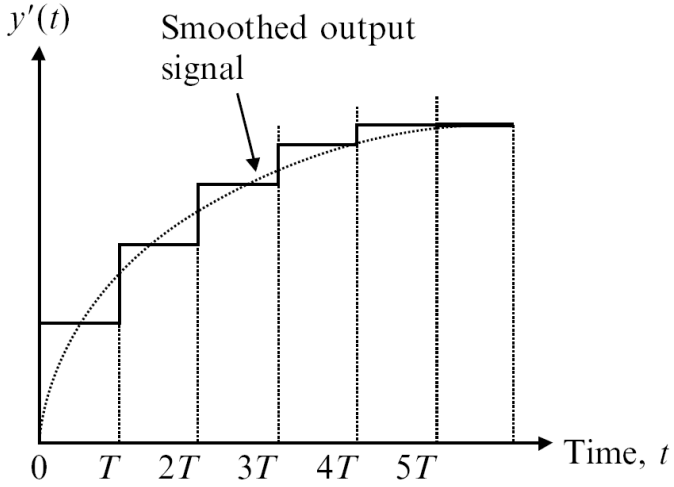
- Usually, the SNR is expressed in dB

$$SQNR = 10\log_{10}\left(3 \cdot 2^{2B} \sigma_x^2\right) =$$
$$= 4.77 + 6.02B + 10\log_{10} \sigma_x^2$$

For each additional bit used in the ADC, the converter provides about 6-dB SNR gain.

DAC & Reconstruction





- Most commercial DACs are zero-order-hold
  - ➔ Convert binary to analog level and hold that value for T second
  - ➔ DAC produces a staircase shape analog waveform
- Reconstruction smoothes the staircase-like output signal generated by the DAC
  - ➔ Lowpass filter (the same of the anti-aliasing filter)

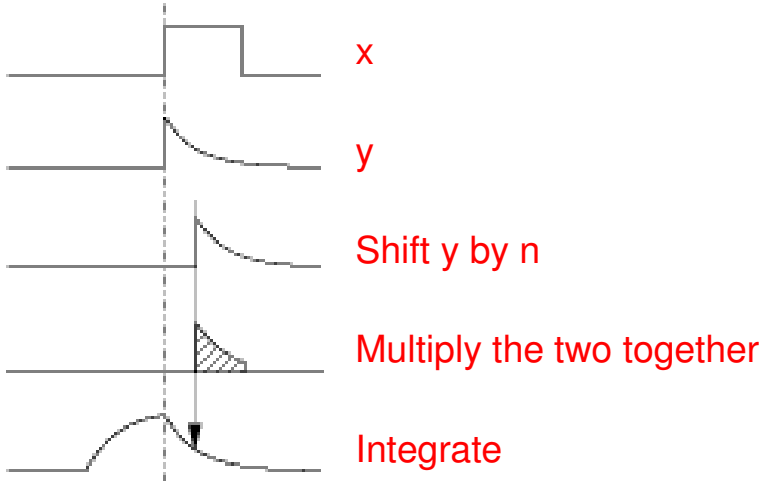
Time Domain Processing

- Correlation
  - ➔ Extract a signal from noise (*autocorrelation*)
  - ➔ Locate a know signal (*cross correlation*)
  - ➔ Identify a signal (*cross correlation*)
- Convolution

Correlation

- Correlation is a weighted moving average

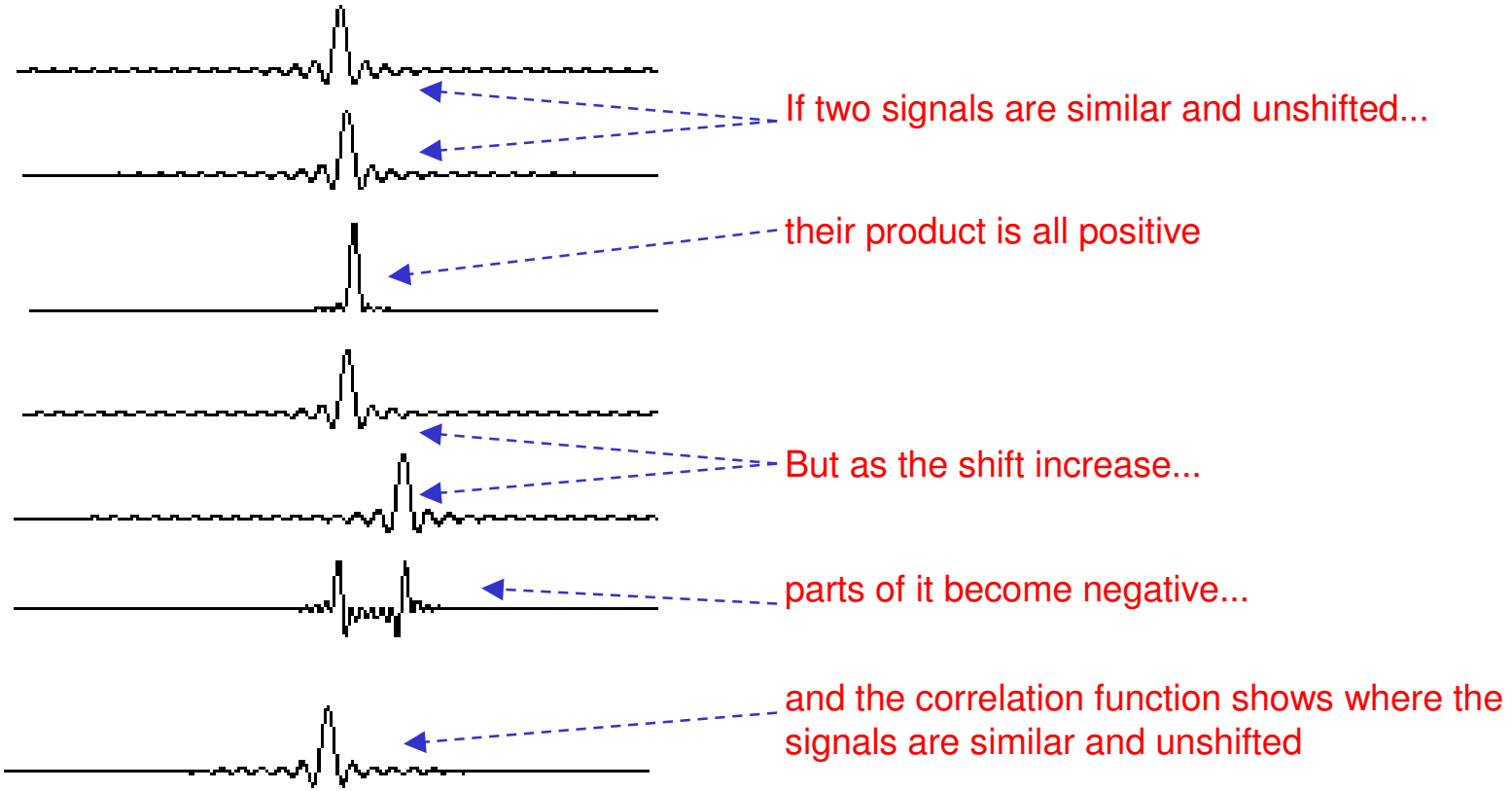
$$r(n) = \sum_k x(k) \times y(k+n)$$



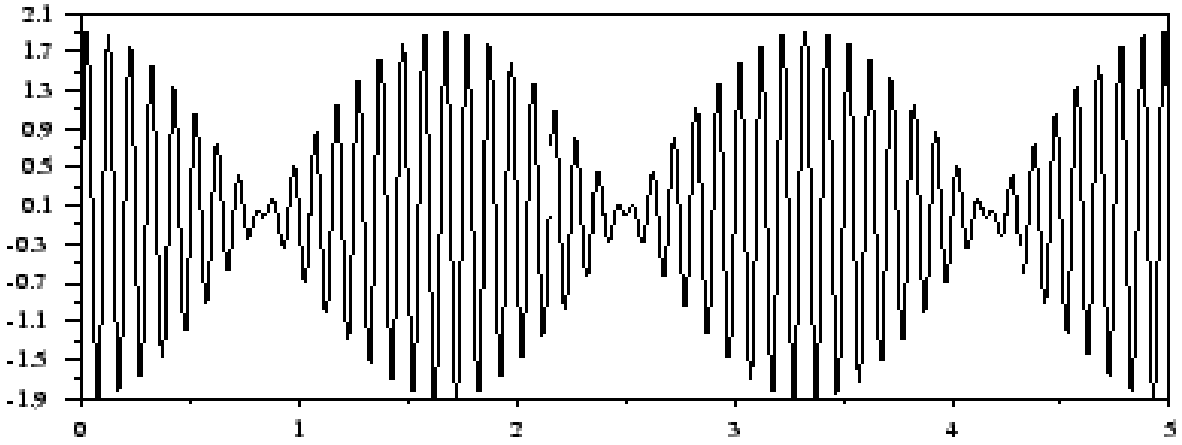
- Requires a lot of calculation
  - ➔ If one signal is of length *M* and the other is of length *N*, then we need (*N* * *M*) multiplications, to calculate the whole correlation function
    - ✓ Note that really, we want to multiply and then accumulate the result - this is typical of DSP operations and is called a *multiply & accumulate* operation

Correlation

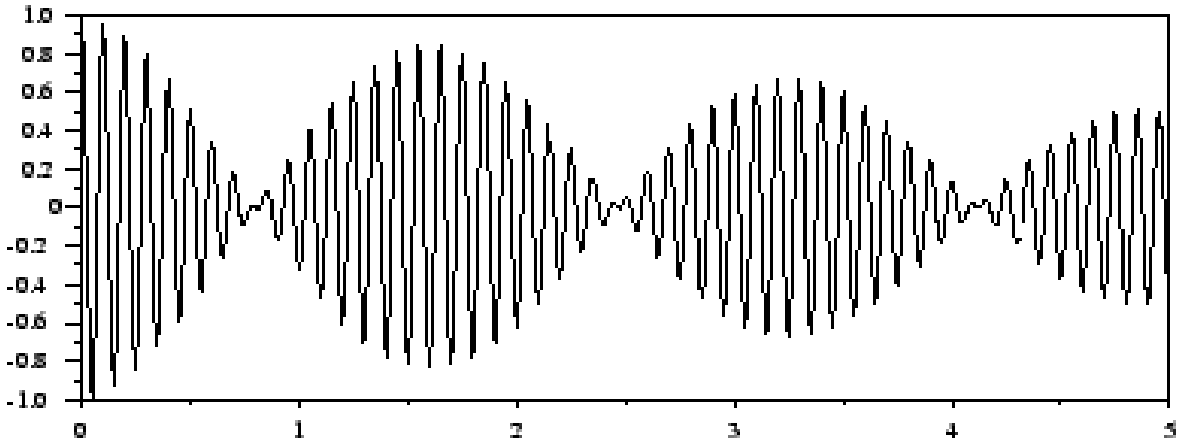
- Correlation has a maximum when two signals are similar in shape
- Correlation is a measure of the similarity between two signals as a function of time shift between them



Detecting Periodicity



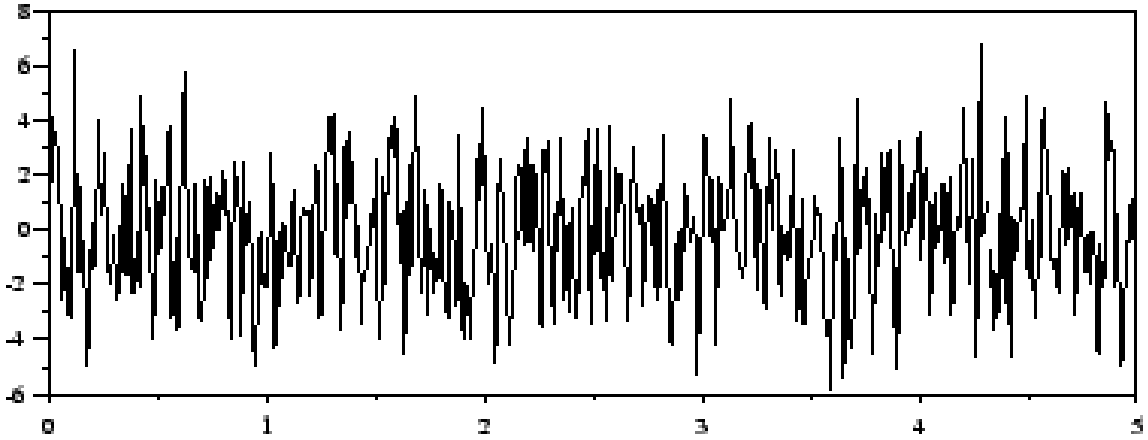
EEG signal



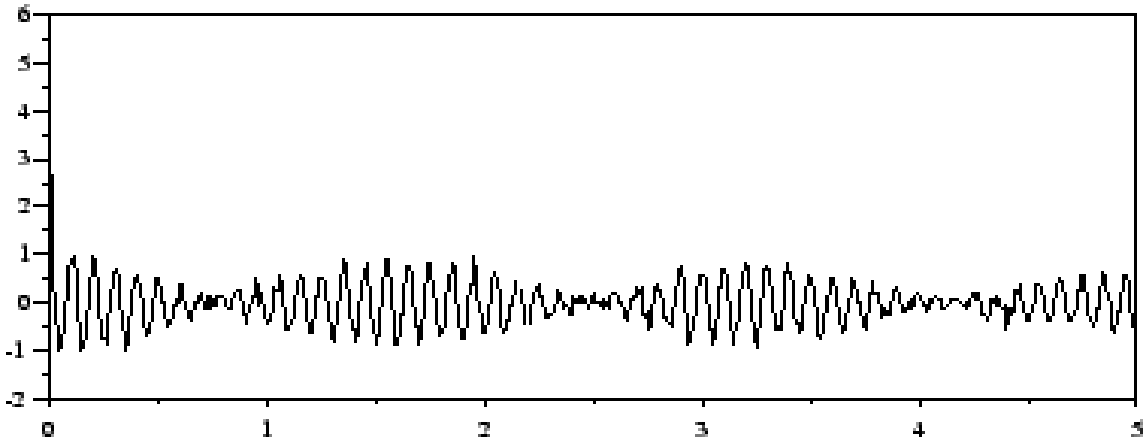
EEG autocorrelation

- Autocorrelation as a way to detect periodicity in signals

Detecting Periodicity



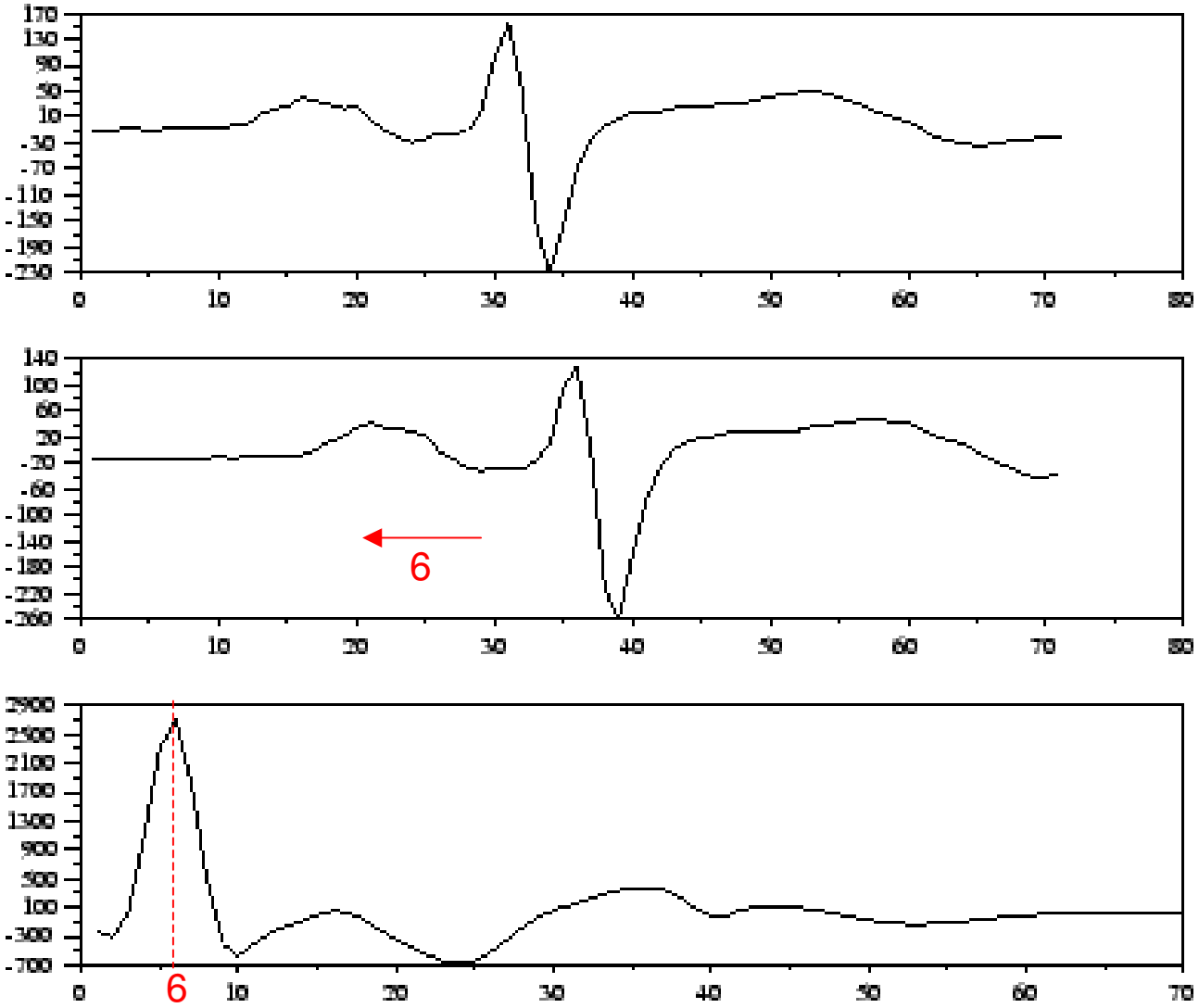
EEG signal with noise



EEG with noise autocorrelation

- Although a rhythm is not even visible (upper trace) it is detected by autocorrelation (lower trace)

Align Signals

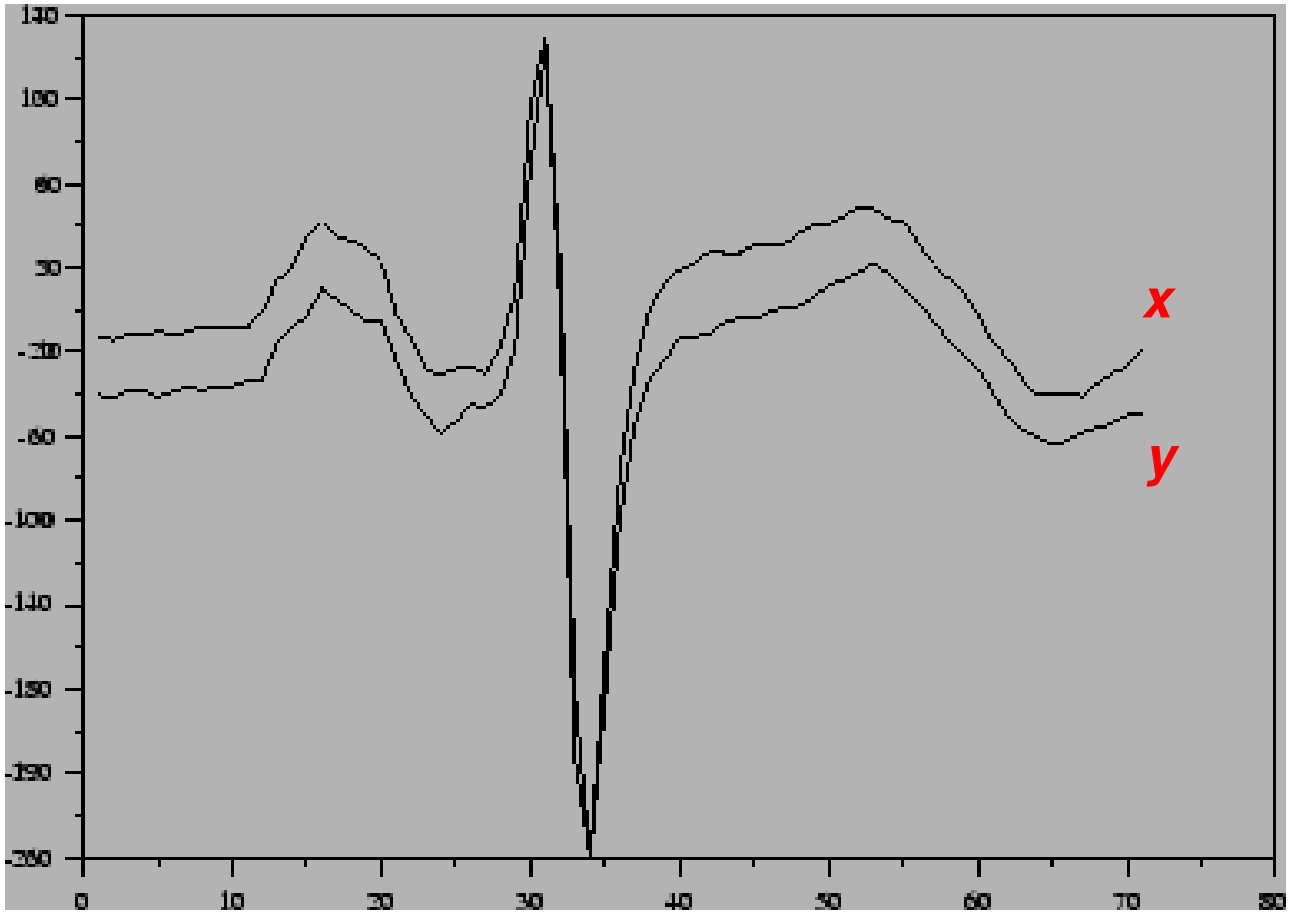


Signal ***x***

Signal ***y***

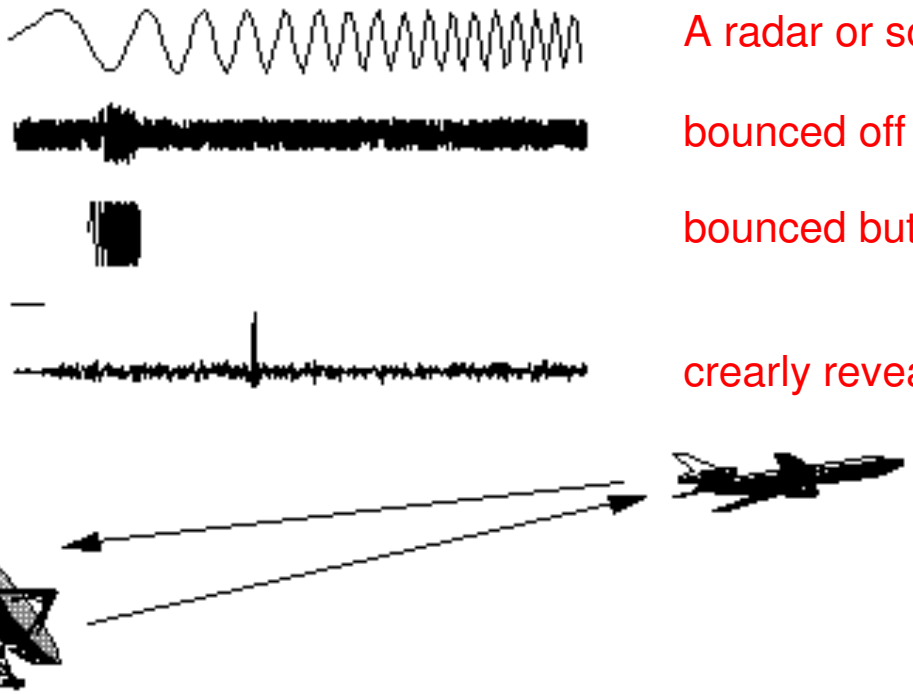
corr(***x***,***y***)

Align Signals



Cross correlation

- Cross correlation (correlating a signal with another) can be used to detect and locate known reference signal in noise



A radar or sonar ‘chirp’ signal

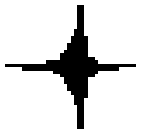
bounced off a target may be buried in noise...

bounced but correlating with the ‘chirp’ reference

crearly reveals when the echo comes

Cross Corelation to Identify a Signal

- Cross correlation (correlating a signal with another) can be used to identify a signal by comparison with a library of known reference signals

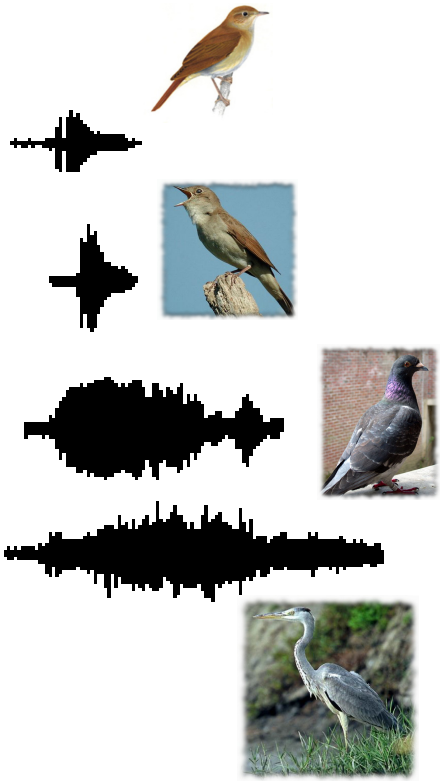


The chirp of a nightingale...

correlates strongly with another nightgale...

but weakly with a dove...

or a heron...



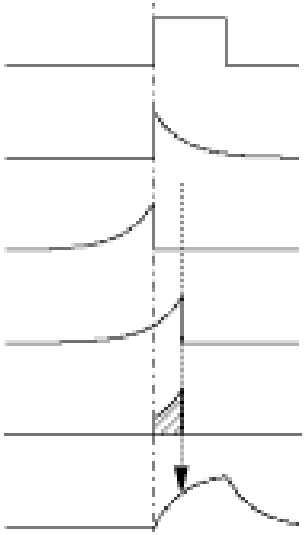
Cross Corelation to Identify a Signal

- Cross correlation is one way in which sonar can identify different types of vessel
  - ➔Each vessel has a unique sonar *signature*
  - ➔The sonar system has a library of pre-recorded echoes from different vessels
  - ➔An unknown sonar echo is correlated with a library of reference echoes
  - ➔The largest correlation is the most likely match

Convolution

- Correlation is a weighted moving average with one signal flipped back to front

$$r(n) = \sum_k x(k) \times y(k-n)$$



To convolve one signal

with another signal

first flip the second signal

Then shift it

Then multiply the two together

And integrate under the curve

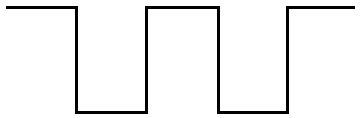
- Requires a lot of calculation
  - ➔ If one signal is of length *M* and the other is of length *N*, then we need (*N* * *M*) multiplications, to calculate the whole convolution function
    - ✓ We need to multiply and then accumulate the result - this is typical of DSP operations and is called a *multiply & accumulate* operation

Convolution vs. Correlation

- Convolution is used for digital filtering
  - ➔ Convolving two signals is equivalent to *multiplying the frequency spectra* of the two signals together
    - ✓ It is easily understood, and is what we mean by filtering
  - ➔ Correlation is equivalent to *multiplying the complex conjugate of the frequency spectrum* of one signal by the frequency spectrum of the other
    - ✓ It is not so easily understood and so convolution is used for digital filtering
- Convolving by multiplying frequency spectra is called *fast convolution*

Fourier Transform

- The *Fourier Transform* is a mathematical procedure that allows to convert a signal from the time domain to the frequency domain
- Any signal or waveform could be made up just by adding together a series of sine waves with appropriate *amplitude* and *phase*



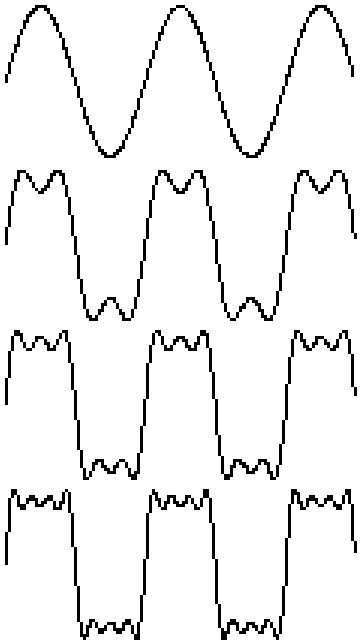
A square wave can be made by adding...

the fundamental

minus 1/3 of the third harmonic

plus 1/5 of the fifth harmonic...

minus 1/7 of the 7th harmonic...



Fourier Transform

- The Fourier transform is an equation to calculate the *frequency*, *amplitude* and *phase* of each sine needed to make up any given signal
  - ➔ The *Fourier Transform* (FT) is a mathematical formula using integrals
  - ➔ The *Discrete Fourier Transform* (DFT) is a discrete numerical equivalent using sums instead of integrals
  - ➔ The *Fast Fourier Transform* (FFT) is just a computationally fast way to calculate the DFT
- The Discrete Fourier Transform involves a summation

$$H(f) = \sum_{k} c(k) \times e^{-2\pi jk(f\Delta)}$$

- DFT and the FFT involve a lot of multiply and accumulate the result
  - ➔ This is typical of DSP operations and is called a *multiply & accumulate* operation

Frequency Spectra

- With some signals it is easy to see that they are composed of different frequencies
  - ➔ A chord played on the piano is obviously made up of the different pure tones generated by the keys pressed
  - ➔ You can use a piano as an acoustic spectrum analyser to show that a hand clap has a frequency spectrum
    - ✓ Open the lid of the piano and hold down the 'loud' pedal
    - ✓ Clap your hands loudly over the piano
    - ✓ You will hear (and see) the strings vibrate to echo the clap sound
      - – The strings that vibrate show the frequencies
      - – The amount of vibration shows the amplitude

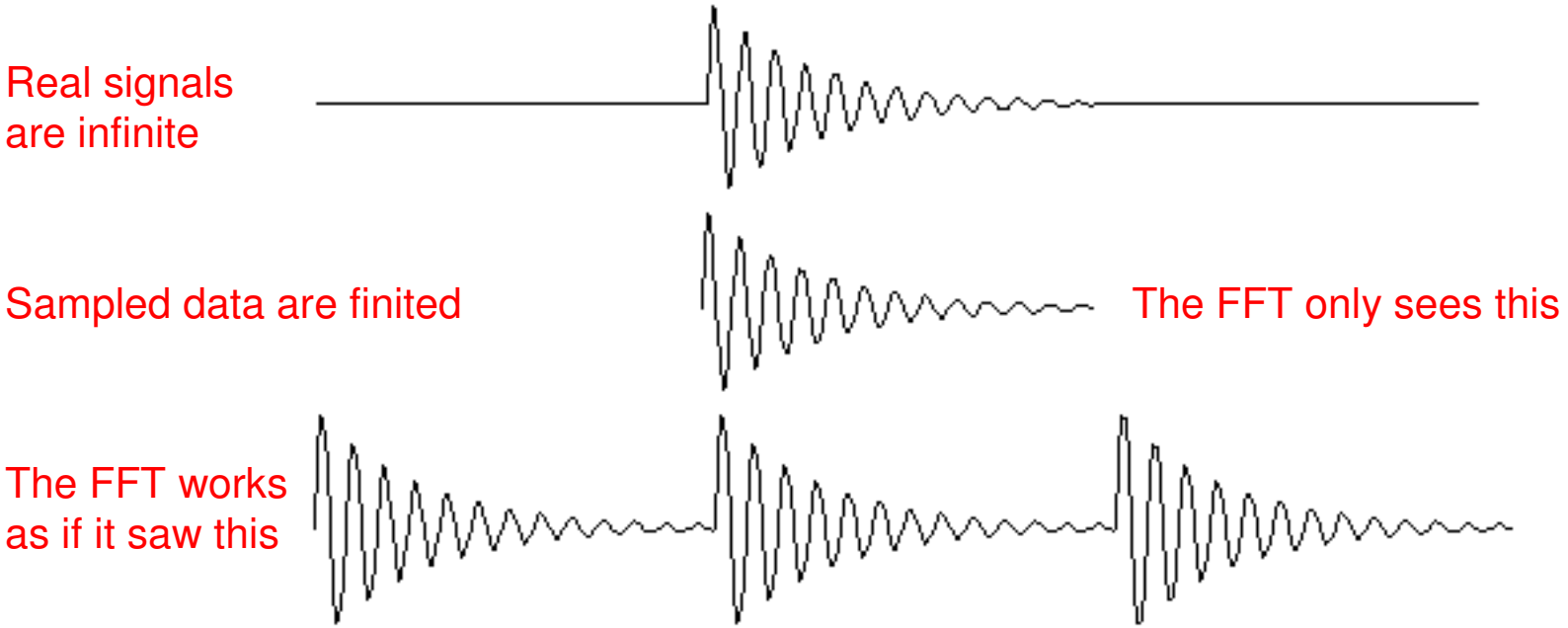
Short Term Fourier Transform

$$H(f) = \sum_{k} c(k) \times e^{-2\pi j k (f\Delta)}$$

- Fourier transform
  - ➔ The signal is analysed over all time
  - ➔ ...an infinite duration
- Short Time Fourier Transform (*STFT*)
  - ➔ Evaluates the way frequency content changes with time

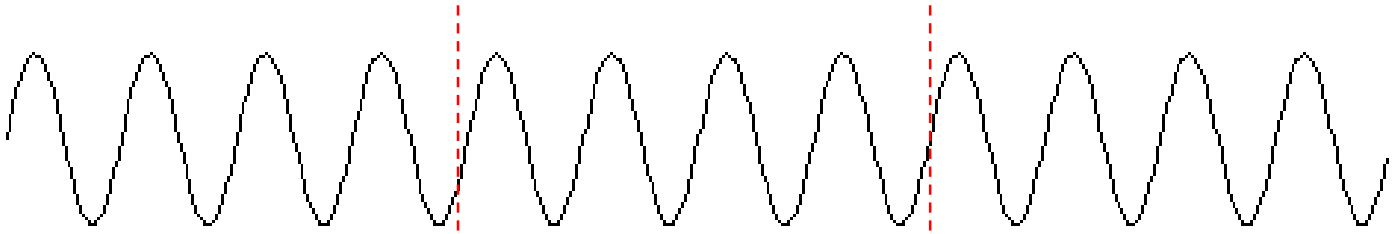
Short Signals

- If we measure the signal for a short time
  - ➔ What happened to the signal before and after we measured it?
  - ➔ FFT makes an assumption about what happened before and after
  - ➔ The FFT assumes the data are periodic for all time



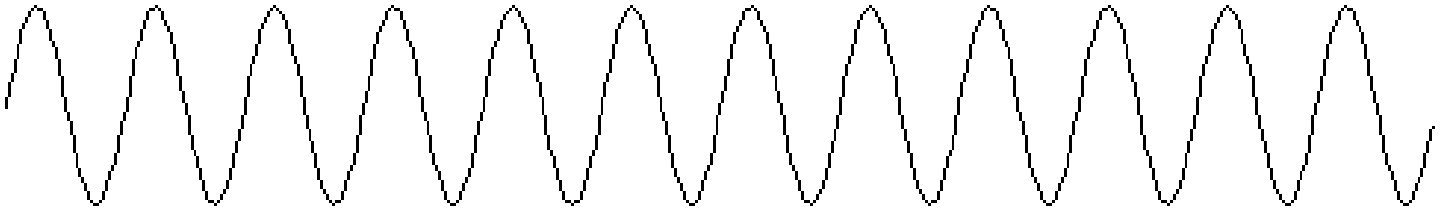
Short Signals

This is
the signal



Period fits the sample time

The FFT works
as if it saw this

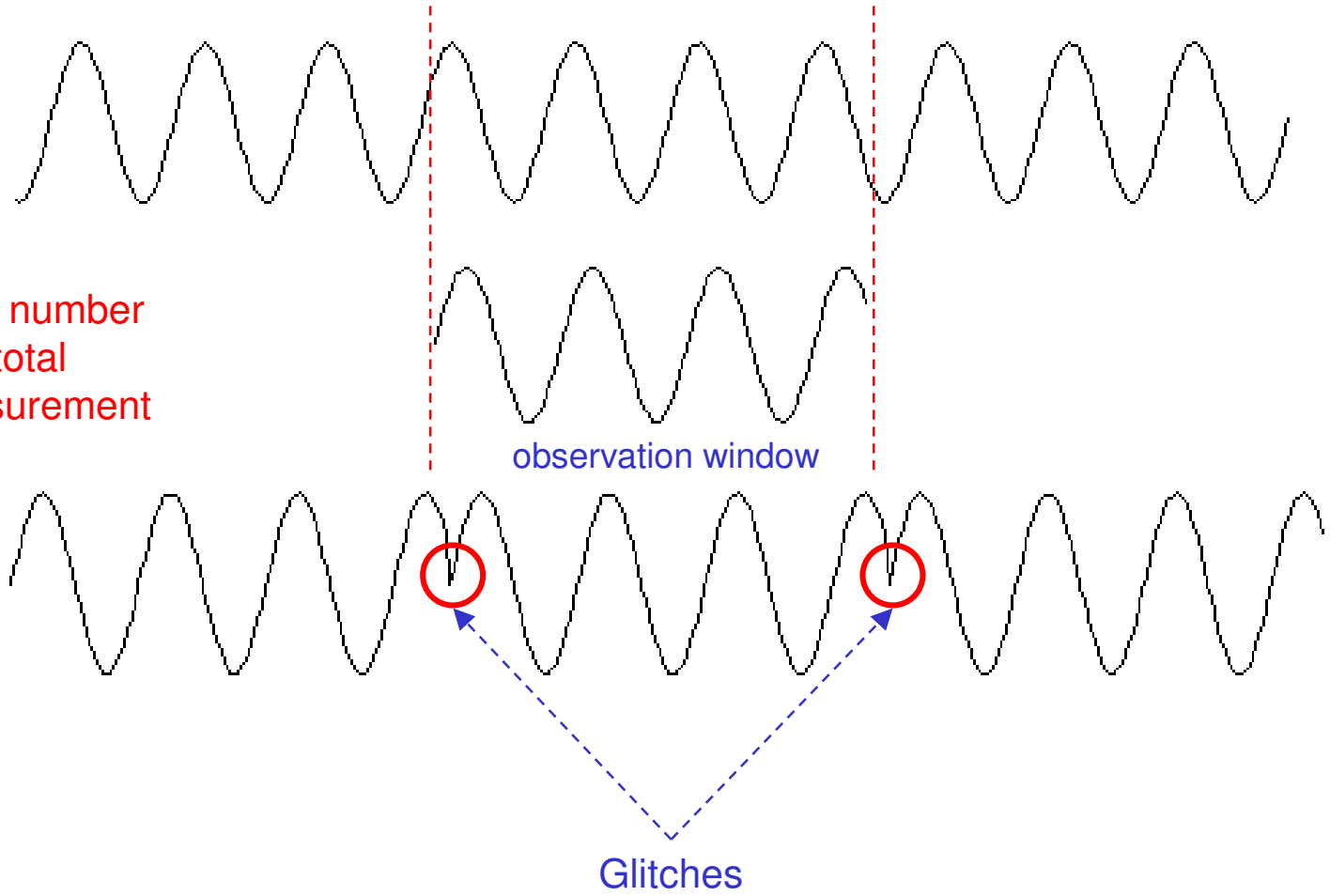


Short Signals

This is
the signal

Not quite an integral number
of cycles fit into the total
duration of the measurement

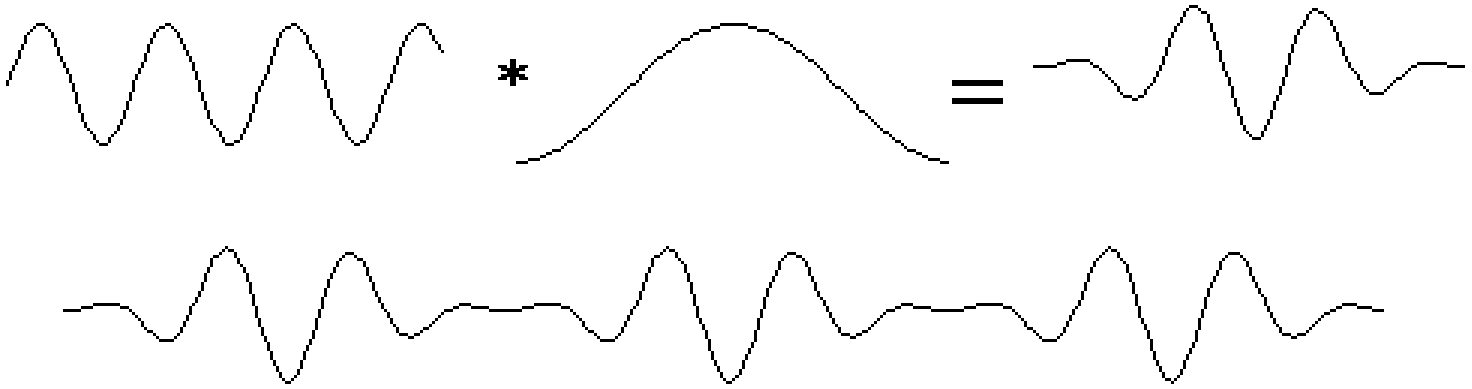
The FFT works
as if it saw this



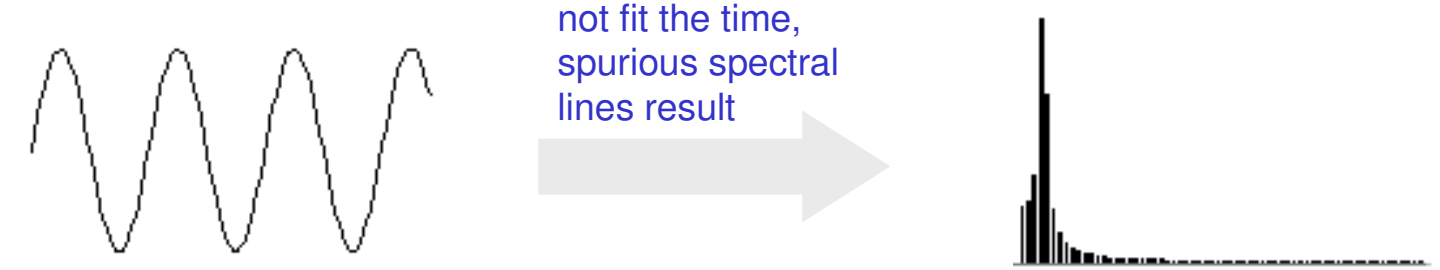
Short Signals

- If the period exactly fits the measurement time
  - ➔ The frequency spectrum is correct
- If the period does not match the measurement time
  - ➔ The frequency spectrum is incorrect - it is broadened
- The size of the glitch depends on when the first measurement occurred in the cycle
  - ➔ The broadening will change if the measurement is repeated

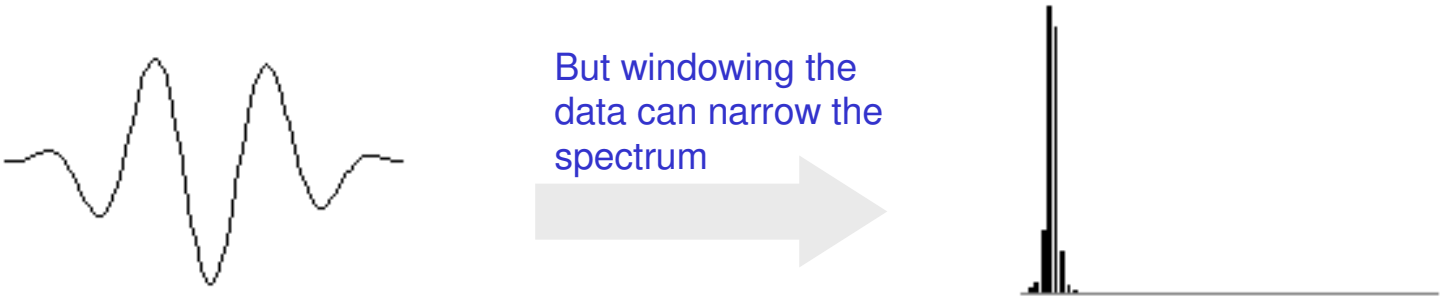
Windowing



If the period does not fit the time, spurious spectral lines result



But windowing the data can narrow the spectrum



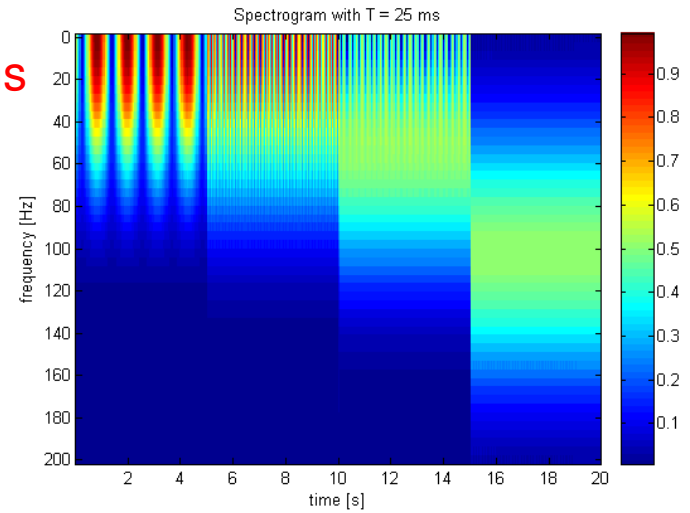
Windowing Tradeoffs (1/2)

- A **wide window** gives better frequency resolution but poor time resolution
- A **narrower window** gives good time resolution but poor frequency resolution

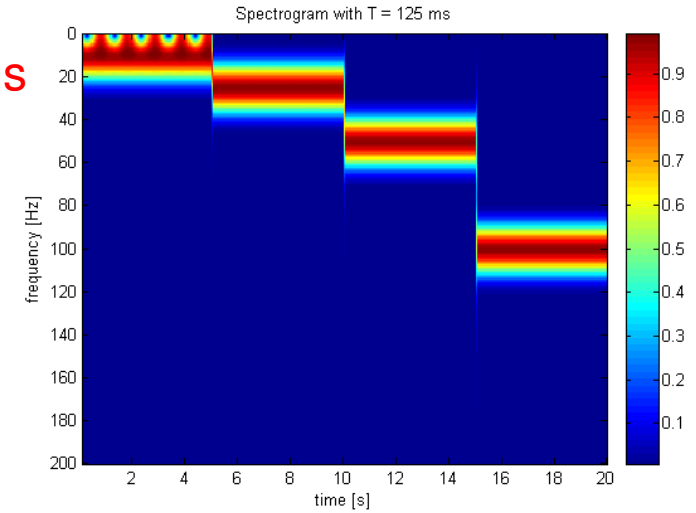
$$x(t) = \begin{cases} \cos(2\pi 10t) & 0 \le t \le 5s \\ \cos(2\pi 25t) & 5 \le t < 10s \\ \cos(2\pi 50t) & 10 \le t < 15s \\ \cos(2\pi 100t) & 15 \le t < 20s \end{cases}$$

Windowing Tradeoffs (1/2)

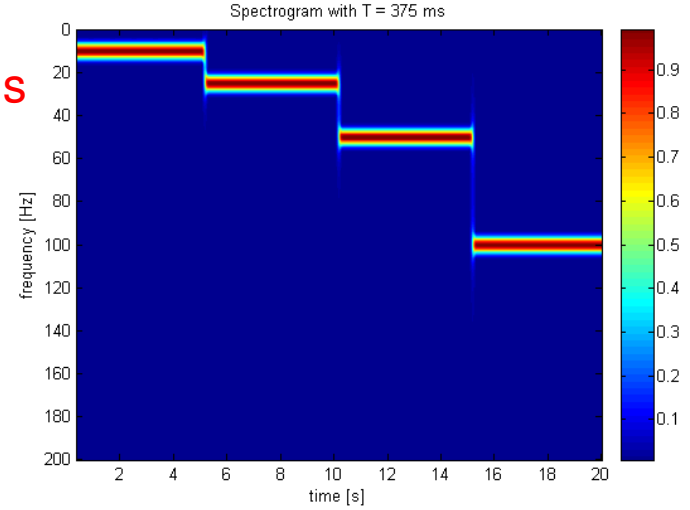
T=25 ms



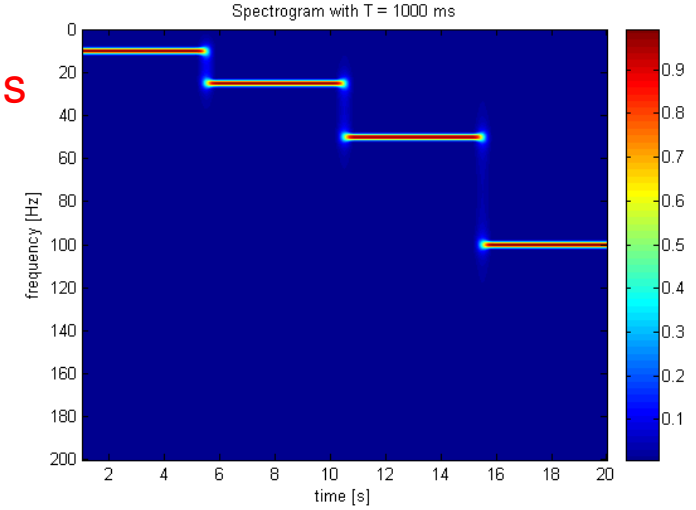
T=125 ms



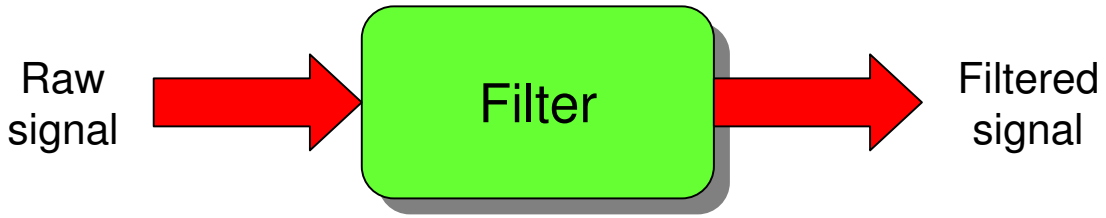
T=375 ms



T=1000 ms



Filtering



- The function of a filter is to remove unwanted parts of the signal
  - ➔ Random noise
  - ➔ Extract useful parts of the signal
    - ✓ Components lying within a certain frequency range
- Filters
  - ➔ Analog
  - ➔ Digital

Analog Filters

- An *analog filter* uses analog electronic circuits
  - ➔Use components such as resistors, capacitors and op amps
- Widely used in such applications
  - ➔Noise reduction
  - ➔Video signal enhancement
  - ➔Graphic equalisers in hi-fi systems
  - ➔..., and many other areas

Digital Filters

- A *digital filter* uses a digital processor to perform numerical calculations on sampled values of the signal
  - ➔ Specialised DSP chip

Advantage of Digital Filters

- Programmability
  - ➔ The digital filter can easily be changed without affecting the circuitry
- Analog filter circuits are subject to drift and are dependent on temperature
- Digital filters can handle low frequency signals accurately
- As the speed of DSP technology continues to increase, digital filters are being applied to high frequency signals in the RF domain
- Versatility
  - ➔ Adapt to changes in the characteristics of the signal

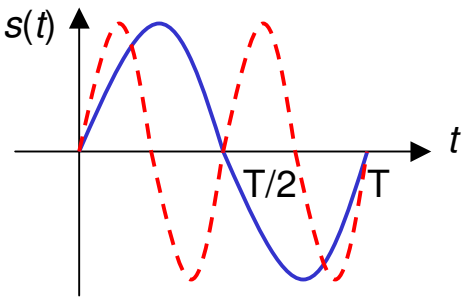
Digital Filter Types & Specifications

- Types
  - Lowpass
  - Highpass
  - Bandpass
  - Bandstop
- Graphical specifications
  - Starting point to determine the coefficients of the digital filter
  - Ideal plot versus the frequency of where the gain curve of the digital filter is allowed to go and not allowed to go

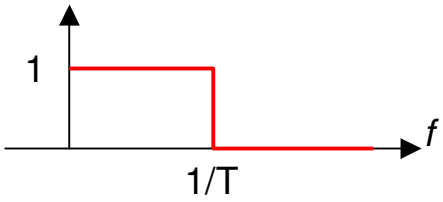
Plot of Gain

$$\text{gain} = 20\ \lg_{10}(\text{magnitude transfer function}) = 20\ \lg_{10} \frac{\text{amplitude of output sinusoid at frequency } \omega}{\text{amplitude of input sinusoid at frequency } \omega}$$

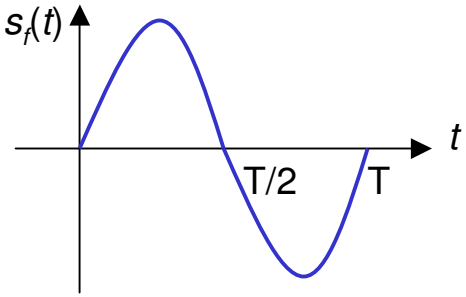
- With a plot of gain vs. frequency of a filter it is easy to see what the filter does to the ouput by multiplying the input amplitude at any frequency by the gain at that frequency



Input signal (red+blue)



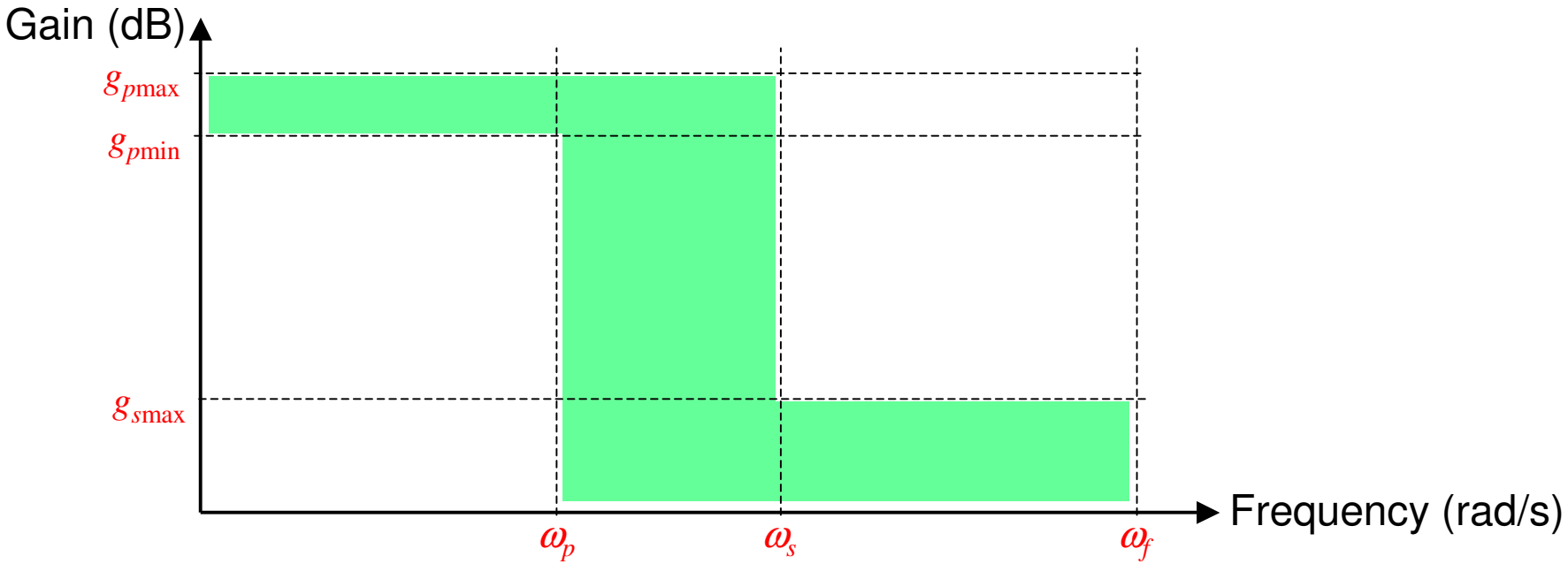
Filter



Output signal

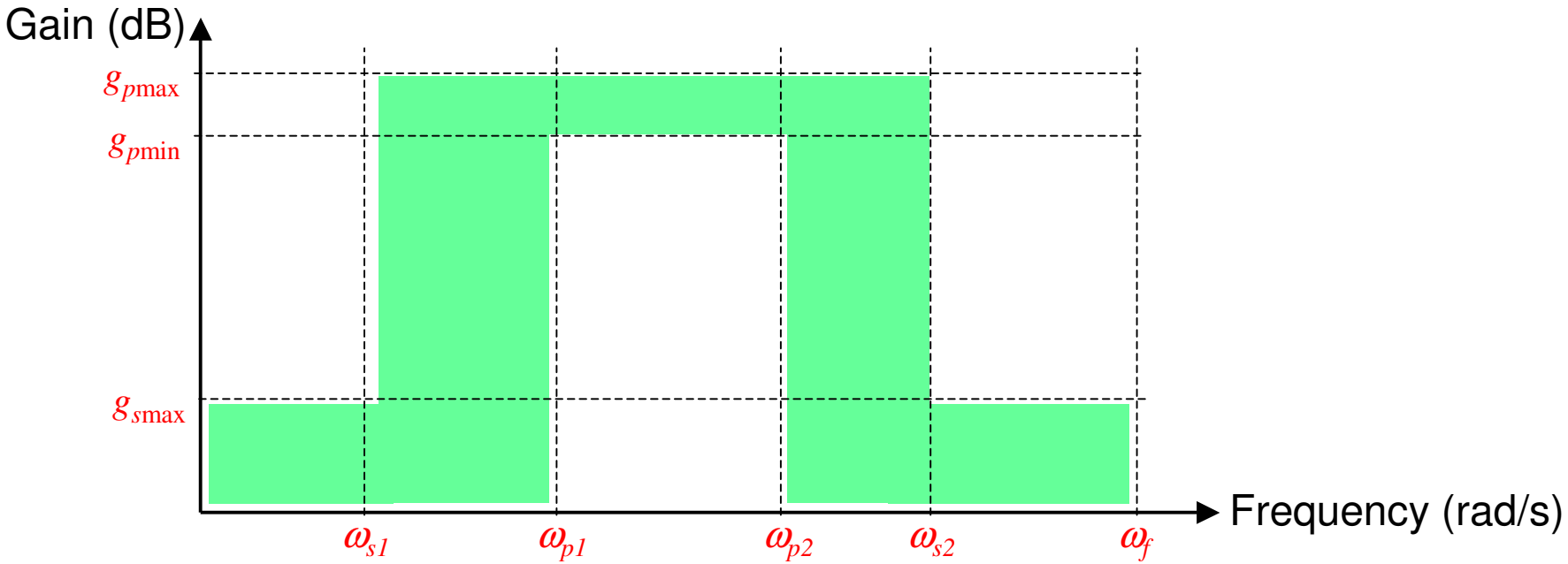
Lowpass Digital Filter Specification

- g_{pmax}/g_{pmin} max/min allowed gain in the passband
- g_{smax} max allowed gain in the stopband
- ω_p highest frequency in the passband
- ω_s lowest frequency in the stopband
- ω_f folding frequency (half of the sampling frequency π/T rad/sec)



Bandpass Digital Filter Specification

- g_{pmax}/g_{pmin} max/min allowed gain in the passband
- g_{smax} max allowed gain in the stopband
- ω_{s1} upper frequency limit of the lower stopband
- ω_{p1} lower frequency limit of the passband
- ω_{p2} upper frequency limit of the passband
- ω_{s2} lower frequency limit of the upper stopband
- ω_f folding frequency (half of the sampling frequency π/T rad/sec)



FIR and IIR Filters

- Finite Impulse Response (*FIR*)
  - ➔ Non-recursive filter

$$y_n = a_0 x_n + a_1 x_{n-1} + a_2 x_{n-2} + \ldots + a_m x_{n-m}$$

  - ➔ Order is m
- Infinite Impulse Response (*IIR*)
  - ➔ Recursive filter

$$y_n = a_0 x_n + a_1 x_{n-1} + a_2 x_{n-2} + \ldots + a_m x_{n-m} + b_1 y_{n-1} + b_2 y_{n-2} + b_3 y_{n-3} + \ldots + b_p y_{n-p}$$

  - ➔ Order is $\max\{m,p\}$

Transfer Function (IIR)

$$b_0 y_n + b_1 y_{n-1} + b_2 y_{n-2} = a_0 x_n + a_1 x_{n-1} + a_2 x_{n-2}$$

- z^{-1} unit delay operator
  - ➔ $z^{-1} x_n = x_{n-1}$
  - ➔ $z^{-2} x_n = x_{n-2}$

$$(b_0 + b_1 z^{-1} + b_2 z^{-2}) y_n = (a_0 + a_1 z^{-1} + a_2 z^{-2}) x_n$$

$$\frac{y_n}{x_n} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}}$$

Transfer function for a second-order recursive (IIR) filter

Transfer Function (FIR)

$$y_n = a_0 x_n + a_1 x_{n-1} + a_2 x_{n-2}$$

- z^{-1} unit delay operator
  - $z^{-1} x_n = x_{n-1}$
  - $z^{-2} x_n = x_{n-2}$

$$y_n = (a_0 + a_1 z^{-1} + a_2 z^{-2}) x_n$$

$$\frac{y_n}{x_n} = a_0 + a_1 z^{-1} + a_2 z^{-2}$$

Transfer function for a second-order non-recursive (FIR) filter

Filter Frequency Response

$$y(n) = \sum_k c(k) \times x(n-k) + \sum_j d(j) \times y(n-j)$$

$$H(f) = \frac{\sum_k c(k) \times e^{-2\pi jk(f\Delta)}}{1 - \sum_k d(k) \times e^{-2\pi jk(f\Delta)}}$$

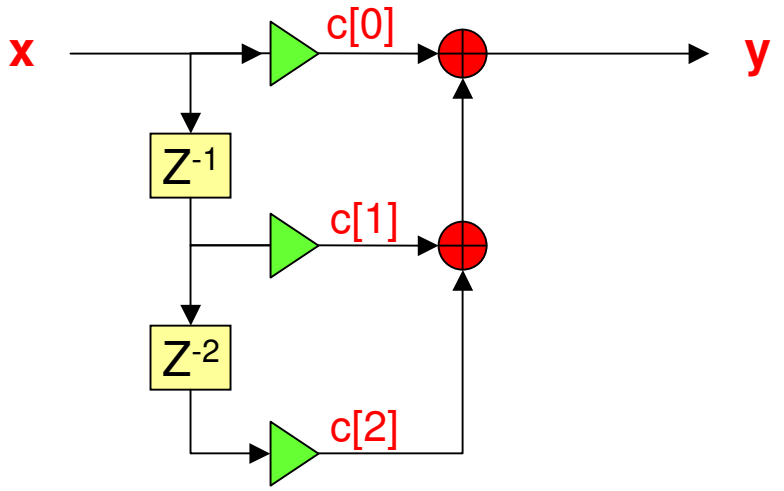
$$z = e^{j\omega\Delta}$$

- When designing a digital filter we want to do the *inverse* operation
  - ➔ Calculate the filter coefficients having first defined the desired frequency response
  - ➔ Additional constraint
    - ✓ We usually want to design a filter that meets the requirement but which requires the least possible amount of computation
      - – Using the smallest number of coefficients

FIR Filters

- The filter equation is simplified by excluding the possibility of feedback

$$y(n) = \sum_k c(k) \times x(n-k)$$



- The filter response is

$$H(f) = \sum_k c(k) \times e^{-2\pi j k (f\Delta)}$$

➔ This frequency response is just the Fourier transform of the filter coefficients

FIR Filters

- So, the coefficients for an FIR filter can be calculated "simply" by taking the *Inverse Fourier Transform* of the desired frequency response
- Here is a recipe for calculating FIR filter coefficients
  - ➔Decide upon the desired frequency response
  - ➔Calculate the inverse Fourier transform
  - ➔Use the result as the filter coefficients
- …BUT...

FIR Filters

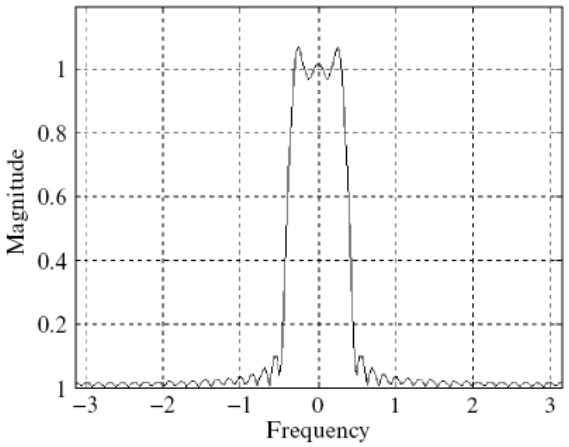
- …BUT…
  - ➔ The iFT has to take samples of the continuous desired frequency response
  - ➔ To define a sharp filter needs closely spaced frequency samples - so a lot of them
  - ➔ So the iFT will give us a lot of filter coefficients
  - ➔ But we don't want a lot of filter coefficients
- A better recipe for calculating FIR filter is:
  - ➔ Specify the desired frequency response using lots of samples
  - ➔ Calculate the inverse iFT
  - ➔ This gives us a lot of filter coefficients
  - ➔ So truncate the filter coefficients to give us less
  - ➔ Then calculate the FT of the truncated set of coefficients to see if it still matches our requirement
- …BUT...

FIR Filters

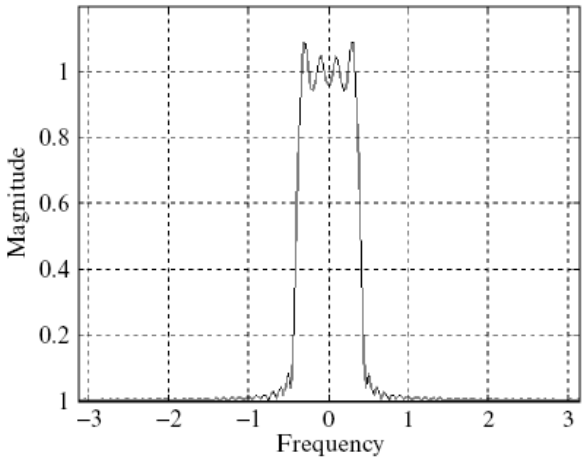
- Truncating the filter coefficients means we have a truncated signal...
- ...And a truncated signal has a broad frequency spectrum
- Applying a window function is a simple way to sharpen up the frequency spectrum of a truncated signal
  - ➔ Specify the desired frequency response using lots of samples
  - ➔ Calculate the inverse Fourier transform
  - ➔ This gives us a lot of filter coefficients
  - ➔ So truncate the filter coefficients to give us less
  - ➔ Apply a window function to sharpen up the filter's frequency response
  - ➔ Then calculate the Fourier transform of the truncated set of coefficients to see if it still matches our requirement
- This is called the window method of FIR filter design

Example

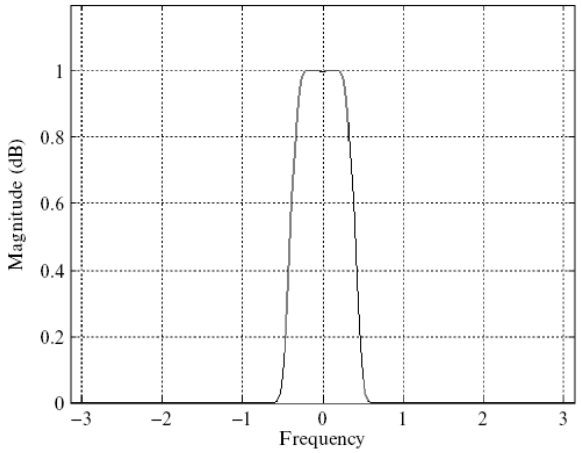
Fourier seies method
40 coefficients



Fourier seies method
60 coefficients

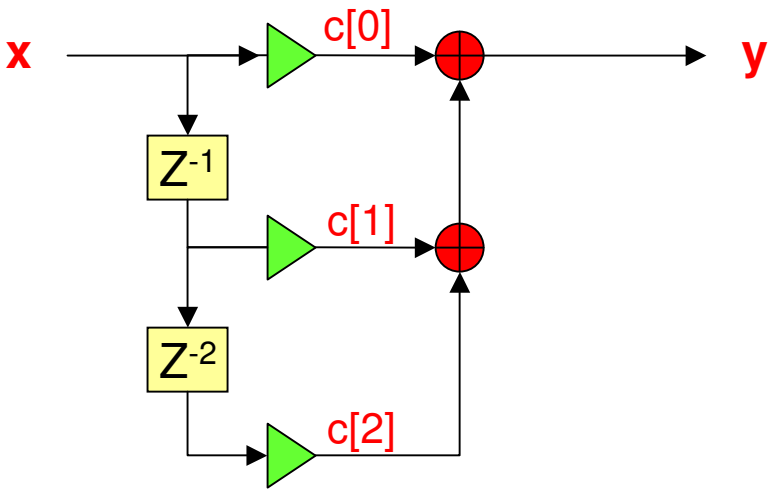


Window method
60 coefficients



Characteristics of DSP Processors

- The basic DSP operations
  - Additions and multiplications
    - Fetch two operands
    - Perform the addition or multiplication (usually both)
    - Store the result or hold it for a repetition
  - Delays
    - Hold a value for later use
  - Array handling
    - Fetch values from consecutive memory locations
    - Copy data from memory to memory



Address Generation

- The ability to generate new addresses efficiently is a characteristic feature of DSP processors
- Usually, the next needed address can be generated during the data fetch or store operation, and with no overhead
- DSP processors have rich sets of address generation operations

*rP	register indirect	read the data pointed to by the address in register rP
*rP++	postincrement	having read the data, postincrement the address pointer to point to the next value in the array
*rP--	postdecrement	having read the data, postdecrement the address pointer to point to the previous value in the array
*rP++rI	register postincrement	having read the data, postincrement the address pointer *by the amount held in register rI* to point to *rI* values further down the array
*rP++rIr	bit reversed	having read the data, postincrement the address pointer to point to the next value in the array, *as if the address bits were in bit reversed order*

Bit Reversed Addressing

- DSPs are tightly targeted to a small number of algorithms
  - ➔ It is surprising that an addressing mode has been specifically defined for just one application (the FFT)

Addresses generated by a radix-2 FFT

0 (000_2)	➔	0 (000_2)
1 (001_2)	➔	4 (100_2)
2 (010_2)	➔	2 (010_2)
3 (011_2)	➔	6 (110_2)
4 (100_2)	➔	1 (001_2)
5 (101_2)	➔	5 (101_2)
6 (110_2)	➔	3 (011_2)
7 (111_2)	➔	7 (111_2)

- Whithout special support such address transformations would
  - ➔ Take an extra memory access to get the new address
  - ➔ Involve a fair amount of logical instructions

Memory Addressing

- As DSP programmers migrate toward larger programs, they are more attracted to compilers
  - ➔ Such compilers are not able to fully exploit such specific addressing modes
  - ➔ DSP community routinely uses library routines
    - ✓ Programmers may benefit even if they write at a high level

Addressing mode	Percent
Immediate	30,02%
Displacement	10,82%
Register indirect	17,42%
Direct	11,99%
Autoincrement, postincrement	18,84%
Autoincrement, preincrement with 16 bit immediate	0,77%
Autoincrement, preincrement with circular addresing	0,08%
Autoincrement, postincrement by contents of AR0	1,54%
Autoincrement, postincrement by contents of AR0, with circular addressing	2,15%
Autodecrement, postdecrement	6,08%

~90%

Compiler for DSPs

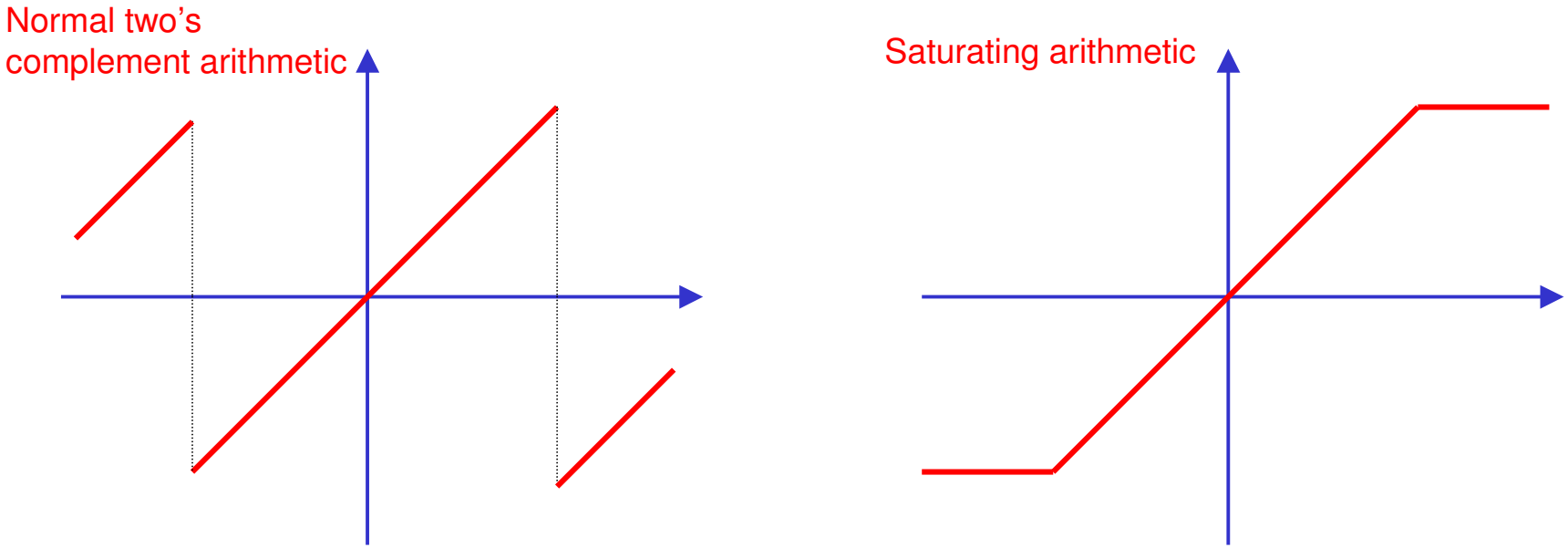
- Despite the well documented advantages in programmer productivity and software maintenance...

TMS320C54 D (C54) for DSPstone kernels	Ratio to assembly in execution time (>1 means slower)	Ratio to assembly in code space (>1 means bigger)
Convolution	11,8	16,5
FIR	11,5	8,7
Matrix 1x3	7,7	8,1
FIR2dim	5,3	6,5
Dot product	5,2	14,1
LMS	5,1	0,7
N real update	4,7	14,1
IIR n biquad	2,4	8,6
N complex update	2,4	9,8
Matrix	1,2	5,1
Complex update	1,2	8,7
IIR one biquad	1,0	6,4
Real update	0,8	15,6
C54 geometric mean	***3,2***	***7,8***

TMS320C6203 (C62) for EEMBC Telecom kernels	Ratio to assembly in execution time (>1 means slower)	Ratio to assembly in code space (>1 means bigger)
Convolution encoder	44,0	0,5
Fixed-point complex FFT	13,5	1,0
Viterbi GSM decoder	13,0	0,7
Fixed-point bit allocation	7,0	1,4
Autocorrelation	1,8	0,7
C62 geometric mean	***10,0***	***0,8***

Saturating Arithmetics

- DSPs are often used in real-time applications
  - ➔ No exception on arithmentic overflow
    - ✓ It could miss an event
  - ➔ To support such an environment, DSP architectures use saturating arithmetic
    - ✓ If the result is too large to be represented, it is set to the largest representable number



The TMS320C3x

- The TMS320C3x generation of DSPs are high performance 32-bit floating-point devices in the TMS320 family
- Extensive internal busing
  - ➔2 operands from memory & 2 operands from the registers file
- Powerful DSP instruction set
- 60 MFLOPS
- High degree of on-chip parallelism
  - ➔Up to 11 operations in a single instruction

General Features

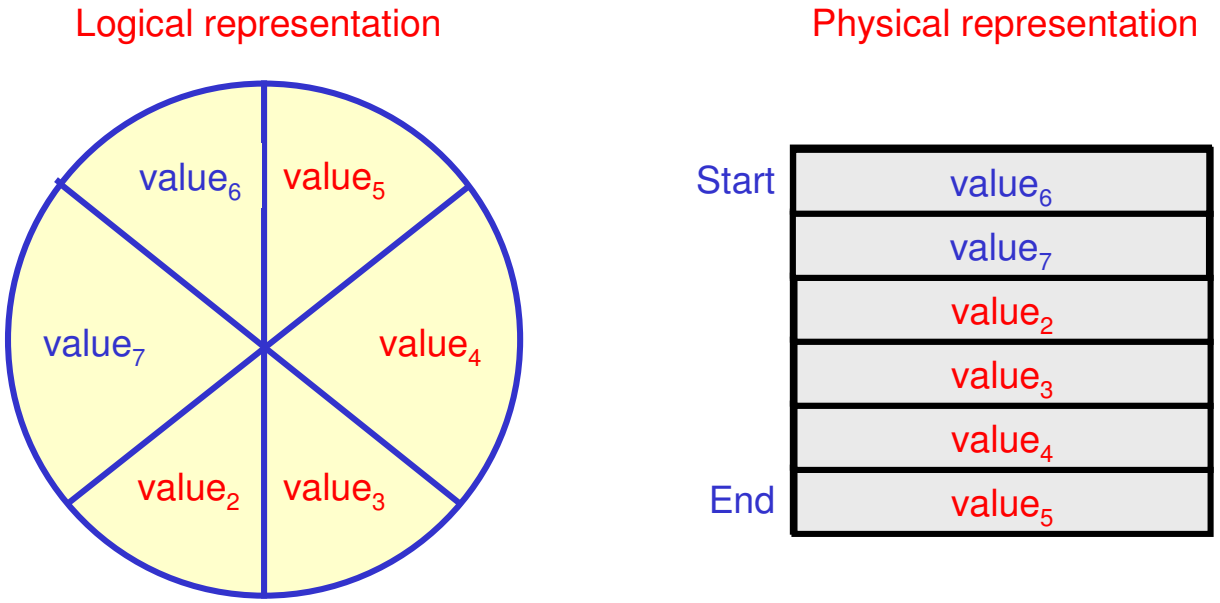
- General-purpose register file
- Program cache
- Dedicated auxiliary register arithmetic units (ARAU)
- Internal dual-access memories
- Direct memory access (DMA)
- Short machine-cycle time

Addressing Modes

- Five types of addressing
  - Register addressing
  - Direct addressing
  - Indirect addressing
  - Immediate addressing
  - PC-relative addressing
- Plus two specialized addressing modes
  - Circular addressing
  - Bit-reverse addressing

Circular Addressing

- Many DSP algorithms, such as convolution and correlation, require a circular buffer in memory
- In convolution and correlation, the circular buffer acts as a sliding window that contains the most recent data to process
- As new data is brought in, the new data overwrites the oldest data



Parallel Operations

- The 13 parallel-operations instructions make a high degree of parallelism possible
- Some of the ’C3x instructions can occur in pairs that are executed in parallel
  - ➔ Parallel loading of registers
  - ➔ Parallel arithmetic operations
  - ➔ Arithmetic/logical instructions used in parallel with a store instruction

Mnemonic	Description
MPYF3 || ADDF3	Multiply and add floating-point value
MPYF3 || SUBF3	Multiply and subtract floating-point value
MPYI3 || ADDI3	Multiply and add integer
MPYI3 || SUBI3	Multiply and subtract integer

…and many many other...

Matrix-Vector Multiplication

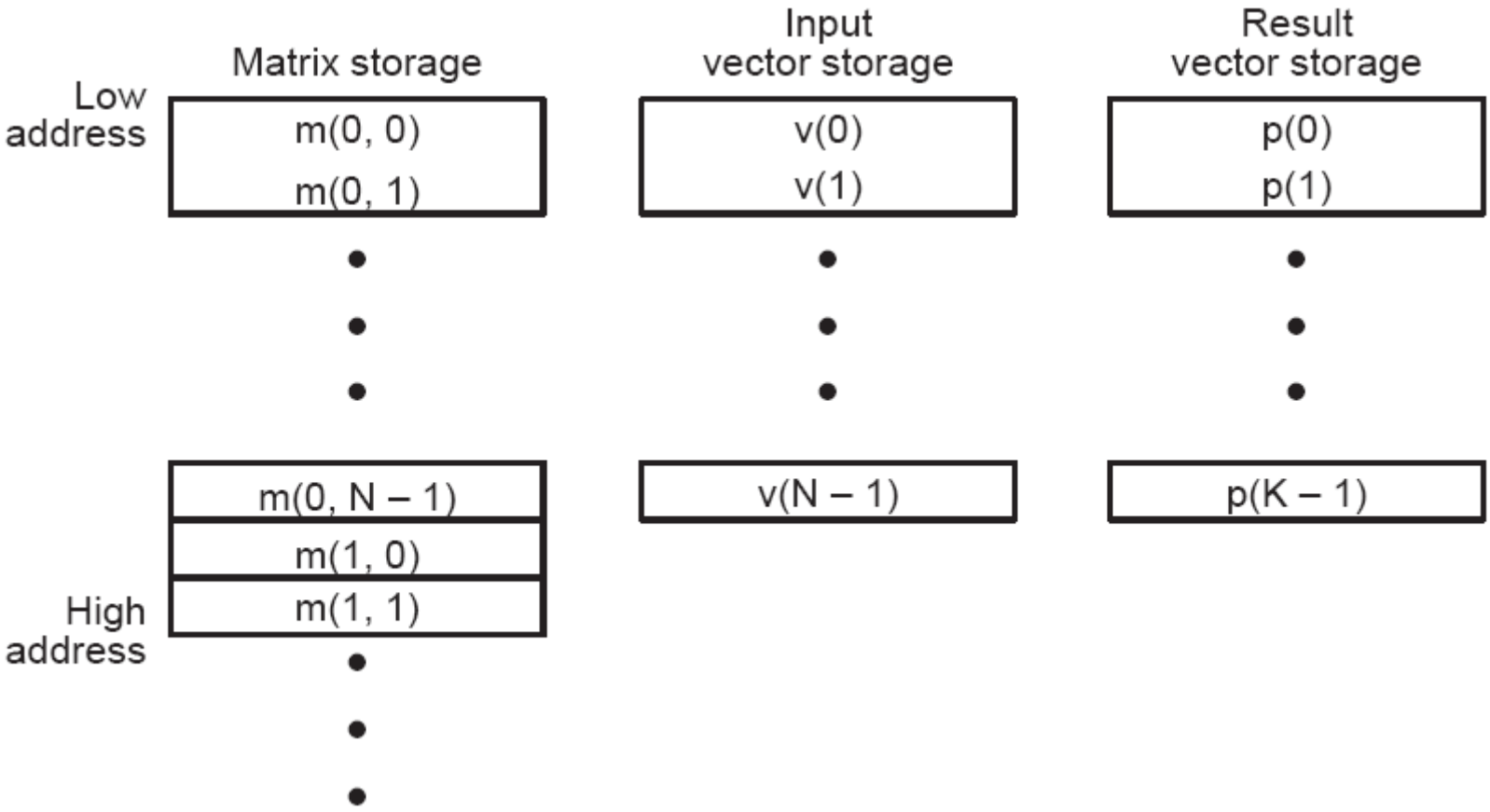
- $[P]_{K\times1}=[M]_{K\times N} \times [V]_{N\times1}$

```
for (i=0; i<K; i++)
{
        p[i] = 0.0;
        for (j=0; j<N; j++)
                p[i] = p[i] + m[i][j] * v[j];
}
```

```
for (i=0; i<K; i++)
{
        p[i] = m[i][0] * v[0];
        for (j=1; j<N; j++)
                p[i] = p[i] + m[i,j] * v[j];
}
```

Matrix-Vector Multiplication

- Data memory organization



Matrix-Vector Multiplication

```
* AR0 : ADDRESS OF M(0,0)
* AR1 : ADDRESS OF V(0)
* AR2 : ADDRESS OF P(0)
* AR3 : NUMBER OF ROWS - 1 (K-1)
* R1  : NUMBER OF COLUMNS - 2 (N-2)

MAT     LDI     R1,IR0              ; Number of columns-2 -> IR0
        ADDI    2,IR0               ; Number of columns -> IR0
ROWS    LDF     0.0,R2              ; Initialize R2
        MPYF3   *AR0++(1),*AR1++(1),R0  ; m(i,0) * v(0) -> R0
        RPTS    R1                  ; Multiply a row by a column
        MPYF3   *AR0++(1),*AR1++(1),R0  ; m(i,j) * v(j) -> R0
     || ADDF3   R0,R2,R2            ; m(i,j-1) * v(j-1) + R2 -> R2
        SUBI    1,AR3
        BNZD    ROWS                ; Counts the no. of rows left
        ADDF    R0,R2               ; Last accumulate
        STF     R2,*AR2++(1)        ; Result -> p(i)
        NOP     *--AR1(IR0)         ; Set AR1 to point to v(0)
```

Delay slot

Summary and Conclusions

- Introduction to Digital Signal Processing
  - Time domain processing
  - Frequency domain processing
  - Digital filters
- Signal Processing using DSP
  - Special features for arithmetic
  - Addressing modes
- More reading
  - Introduction to DSP
    - ✓ http://www.bores.com
  - TMS320C3x General-Purpose Applications User's Guide
    - ✓ http://focus.ti.com/general/docs/tecdocs.tsp