Multi-Objective Optimization

Introduction to MOO

- Most real-world engineering optimization problems are multi-objective in nature
- Objectives are often conflicting
  - Performance vs. Silicon area
  - Quality vs. Cost
  - Efficiency vs. Portability
  - …
- The notion of *optimum* has to be re-defined

Statement of the Problem

- Multiobjective optimization (multicriteria, multiperformance, vector optimization)
  - Find a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions
  - Objectives are usually in conflict with each other
  - **Optimize**: finding solutions which would give the values of all the objective functions acceptable to the designer

Mathematical Formulation

- Find the vector

$$\bar{x} = \left[x_1, x_2, \ldots, x_n \right]$$

- Which will satisfy the *m* inequality constraints

$$g_i(x) \geq 0 \quad i = 1,2,\ldots,m$$

- The *p* equality constraints

$$h_i(x) = 0 \quad i = 1,2,\ldots,p$$

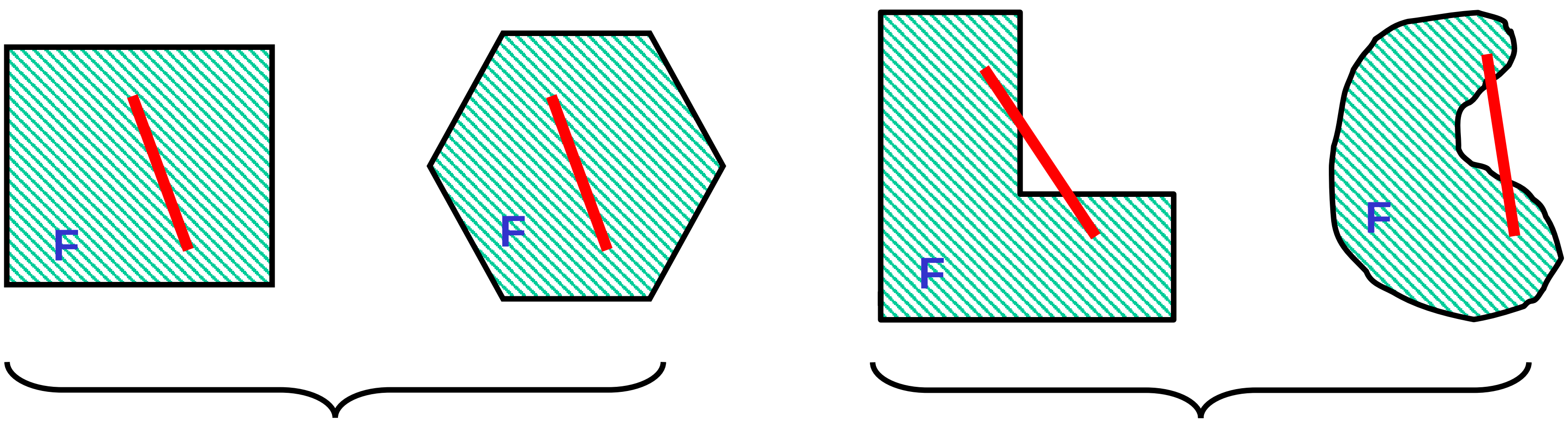
- And optimizes the vector function

$$\bar{f}(\bar{x}) = \left[f_1(\bar{x}), f_2(\bar{x}), \ldots, f_k(\bar{x}) \right]$$

Feasible Region

$$\left.\begin{aligned} g_i(x) &\geq 0 \quad i = 1,2,\ldots,m \\ h_i(x) &= 0 \quad i = 1,2,\ldots,p \end{aligned}\right\}$$

Define the *feasible region F*



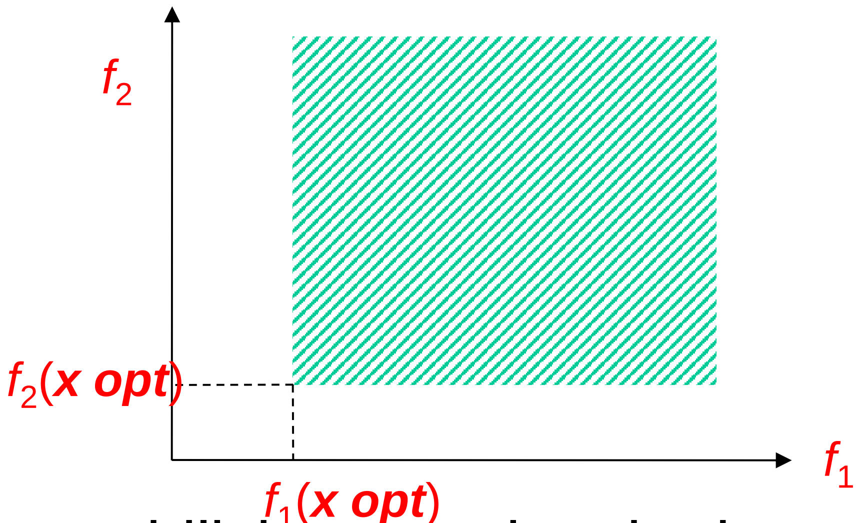
Convex sets

Non-convex sets

Meaning of Optimum

- We rarely have an **x optimum** such that

$$f_i\left(\overline{x}^{opt}\right) \le f_i\left(\overline{x}\right) \quad \forall\ \overline{x} \in F,\ i = 1,2,\ldots,k$$



- We have to estabilish a certain criteria to determine what would be considered as an optimal solution

Pareto Set

- A solution ***x*** ***oϕ*** *F* is said to dominate ***y*** ***oϕ*** *F* if
  - ➔ ***x*** is better or equal to ***y*** in all attributes
  - ➔ ***x*** is strictly better than ***y*** in at least one attribute

Vilfredo Pareto 1848-1923

- Formally, ***x*** dominate ***y***

$$f_i(x) \le f_i(y), \quad i = 1,2,\ldots,k$$

$$\exists\ j \in \{1,2,\ldots,k\} : f_j(x) < f_j(y)$$

- The *Pareto set* consists of solutions that are not dominated by any other solutions

Pareto Front