Introduction to MOO

- Most real-world engineering optimization problems are multi-objective in nature
- Objectives are often conflicting
  - Performance vs. Silicon area
  - Quality vs. Cost
  - Efficiency vs. Portability
  - …
- The notion of *optimum* has to be re-defined

Statement of the Problem

- Multiobjective optimization (multicriteria, multiperformance, vector optimization)
  - Find a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions
  - Objectives are usually in conflict with each other
  - **Optimize**: finding solutions which would give the values of all the objective functions acceptable to the designer

Mathematical Formulation

- Find the vector

$$\bar{x} = [x_1, x_2, \ldots, x_n]$$

- Which will satisfy the *m* inequality constraints

$$g_i(\bar{x}) \geq 0 \quad i = 1, 2, \ldots, m$$

- The *p* equality constraints

$$h_i(\bar{x}) = 0 \quad i = 1, 2, \ldots, p$$

- And optimizes the vector function

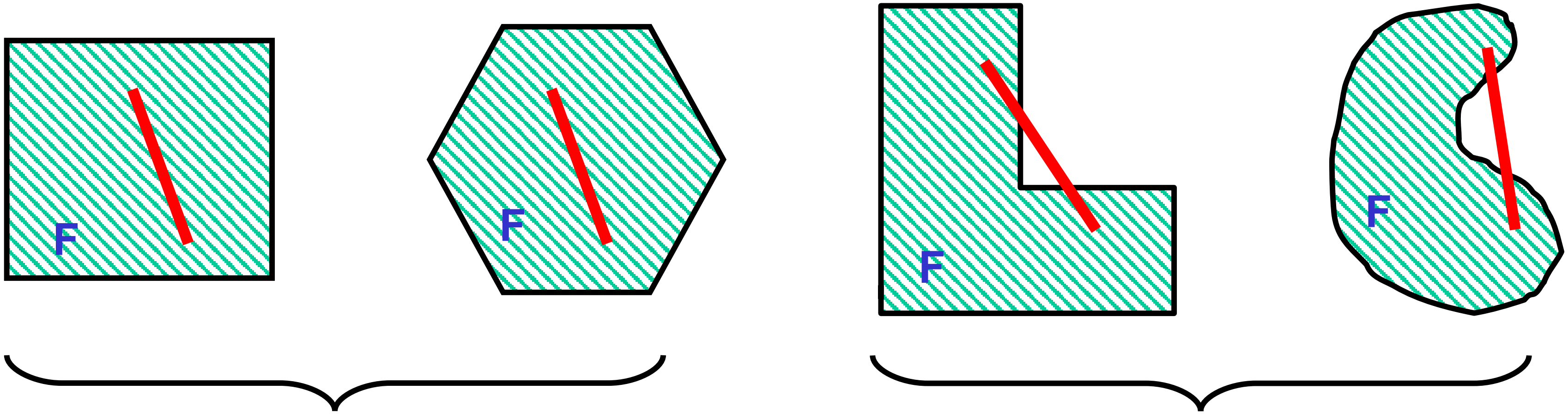
$$\bar{f}(\bar{x}) = [f_1(\bar{x}), f_2(\bar{x}), \ldots, f_k(\bar{x})]$$

Feasible Region

$$g_i(\bar{x}) \geq 0 \quad i = 1, 2, \ldots, m$$

$$h_i(\bar{x}) = 0 \quad i = 1, 2, \ldots, p$$

Define the *feasible region F*



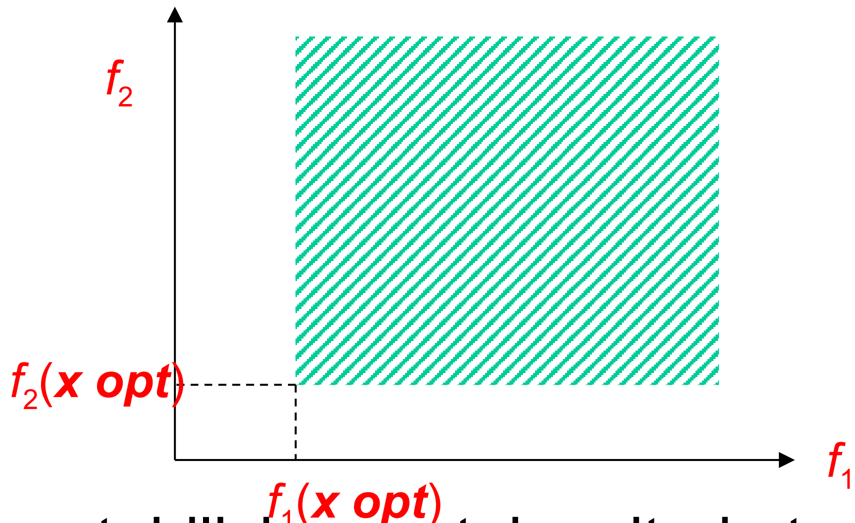
Convex sets

Non-convex sets

Meaning of *Optimum*

- We rarely have an **x optimum** such that

$$f_i\left(\bar{x}^{opt}\right) \le f_i\left(\bar{x}\right) \quad \forall \ \bar{x} \in F,\ i = 1,2,\ldots,k$$



- We have to estabilish a certain criteria to determine what would be considered as an optimal solution

Pareto Set

- A solution ***x*** *oϕ F* is said to dominate ***y*** *oϕ F* if
  - ➔ ***x*** is better or equal to ***y*** in all attributes
  - ➔ ***x*** is strictly better than ***y*** in at least one attribute

Vilfredo Pareto 1848-1923

- Formally, ***x*** dominate ***y***

$$f_i(\bar{x}) \leq f_i(\bar{y}), \quad i=1,2,\ldots,k$$

$$\exists\ j \in \{1,2,\ldots,k\} : f_j(\bar{x}) < f_j(\bar{y})$$

- The *Pareto set* consists of solutions that are not dominated by any other solutions

Pareto Front