

Trombe elettromagnetiche

12.1 - Antenne piramidali

Si abbia una guida d'onda rettangolare eccitata nel modo dominante TE_{01} ($b_g > a_g$). Se essa è connessa ad una struttura metallica a contorno chiuso e a sezione divergente si forma un'antenna piramidale molto usata nella regione delle microonde.

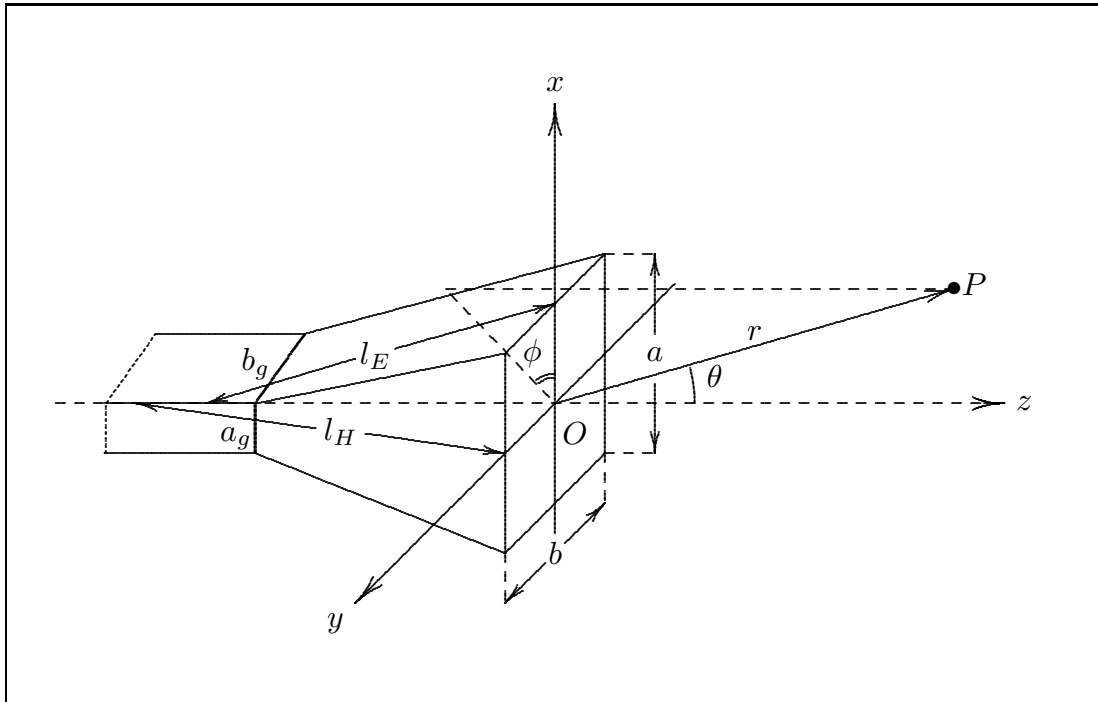


fig.12.1-1

Per conoscere il campo irradiato in un punto P da tale tipo di antenna, bisogna conoscere il campo sull'apertura radiante; per questo si rende necessario lo studio della propagazione in trombe elettromagnetiche. È utile anticipare che il campo dentro strutture a tromba può essere espresso in termini di funzioni d'onda cilindriche che includono le funzioni di Hankel. Il procedimento non è difficile ma è laborioso.

Da tale studio segue che: se i campi della guida di alimentazione sono quelli del modo dominante TE_{01} o TE_{10} e se la lunghezza della tromba è grande rispetto alle dimensioni dell'apertura (per lunghezza della tromba si deve intendere l_E e l_H definite come in figura 12.1-1), la componente del campo elettrico sull'apertura d'uscita ha la stessa forma della distribuzione dell'ampiezza del campo elettrico del modo TE_{01} della guida rettangolare con una variazione di fase parabolica.

Si ha, cioè, sull'apertura:

$$E_{ax}(x, y) = E_0 \text{rect}_{a,b}(x, y) \cos\left(\frac{\pi y}{b}\right) \exp\left\{-\frac{ik}{2}\left(\frac{x^2}{l_E} + \frac{y^2}{l_H}\right)\right\} \quad (12.1.1)$$

Lo spettro angolare descrivente il campo di radiazione è dato dalla (8.13.2) cioè:

$$F_x(\alpha, \beta) = \frac{1}{\lambda^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_{ax}(x, y) \exp \{ ik(\alpha x + \beta y) \} dx dy \quad (8.13.2)$$

Sostituendo la (12.1.1) nella (8.13.2) segue:

$$F_x(\alpha, \beta) = \frac{E_0}{2\lambda^2} \int_{-\frac{a}{2}}^{+\frac{a}{2}} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \left[\exp \left(\frac{i\pi y}{b} \right) + \exp \left(-\frac{i\pi y}{b} \right) \right] \exp \left[-\frac{ik}{2} \left(\frac{x^2}{l_E} + \frac{y^2}{l_H} \right) \right] \cdot \exp \{ ik(\alpha x + \beta y) \} dx dy \quad (12.1.2)$$

$$F_x(\alpha, \beta) = \frac{E_0}{2\lambda^2} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \left[\exp \left(\frac{i\pi y}{b} \right) + \exp \left(-\frac{i\pi y}{b} \right) \right] \exp \left[-\frac{ik}{2} \left(\frac{y^2}{l_H} \right) \right] \exp \{ ik(\beta y) \} dy \cdot \int_{-\frac{a}{2}}^{+\frac{a}{2}} \exp \left[-\frac{ik}{2} \left(\frac{x^2}{l_E} \right) \right] \exp \{ ik(\alpha x) \} dx \quad (12.1.3)$$

Calcoliamo separatamente i due integrali che figurano nella (12.1.3)

$$\int_{-\frac{a}{2}}^{+\frac{a}{2}} \exp \left[-\frac{ik}{2} \left(\frac{x^2}{l_E} \right) \right] \exp \{ ik(\alpha x) \} dx = \int_{-\frac{a}{2}}^{+\frac{a}{2}} \exp \left[-i \left(\frac{kx^2}{2l_E} - k\alpha x \right) \right] dx$$

Osserviamo che:

$$\frac{kx^2}{2l_E} - k\alpha x = \frac{k}{2l_E} (x^2 - 2l_E\alpha x) \quad (12.1.4)$$

Dentro l'espressione fra parentesi nella (12.1.4), aggiungiamo e sottraiamo la quantità $l_E^2\alpha^2$. Si ha:

$$\frac{kx^2}{2l_E} - k\alpha x = \frac{k}{2l_E} (x^2 - 2l_E\alpha x + l_E^2\alpha^2 - l_E^2\alpha^2) = \frac{k}{2l_E} (x - l_E\alpha)^2 - \frac{kl_E}{2}\alpha^2 \quad (12.1.5)$$

Pertanto:

$$\int_{-\frac{a}{2}}^{+\frac{a}{2}} \exp \left[-\frac{ik}{2} \left(\frac{x^2}{l_E} \right) \right] \exp \{ ik(\alpha x) \} dx = \exp \left(i \frac{kl_E\alpha^2}{2} \right) \int_{-\frac{a}{2}}^{+\frac{a}{2}} \exp \left[-i \frac{k}{2l_E} (x - l_E\alpha)^2 \right] dx$$

che si può trasformare in integrale di Fresnel con la posizione $u = \sqrt{\frac{2}{\lambda l_E}} (x - l_E\alpha) \implies$

$dx = \sqrt{\frac{\lambda l_E}{2}} du$. Ne segue:

$$\int_{-\frac{a}{2}}^{+\frac{a}{2}} \exp \left[-\frac{ik}{2} \left(\frac{x^2}{l_E} \right) \right] \exp \{ ik(\alpha x) \} dx = \sqrt{\frac{\lambda l_E}{2}} \exp \left(i \frac{\pi\alpha^2 l_E}{\lambda} \right) \int_{u_1}^{u_2} \exp \left\{ -i \frac{\pi u^2}{2} \right\} du \quad (12.1.6)$$

essendo

$$\begin{aligned} u_2 &= \left\{ \sqrt{\frac{2}{\lambda l_E}} \frac{a}{2} - \frac{2\alpha}{\lambda} \sqrt{\frac{\lambda l_E}{2}} \right\} \\ u_1 &= - \left\{ \sqrt{\frac{2}{\lambda l_E}} \frac{a}{2} + \frac{2\alpha}{\lambda} \sqrt{\frac{\lambda l_E}{2}} \right\} \end{aligned} \quad (12.1.7)$$

L'integrale al secondo membro della (12.1.6) è l'integrale di Fresnel, pertanto:

$$\int_{-\frac{a}{2}}^{+\frac{a}{2}} \exp \left[-\frac{ik}{2} \left(\frac{x^2}{l_E} \right) \right] \exp \{ ik(\alpha x) \} dx = \sqrt{\frac{\lambda l_E}{2}} \exp \left(i \frac{\pi \alpha^2 l_E}{\lambda} \right) \{ \mathcal{F}(u_2) + \mathcal{F}(-u_1) \} \quad (12.1.8)$$

Analogamente calcoliamo il primo integrale della (12.1.3), si ha:

$$\begin{aligned} & \int_{-\frac{b}{2}}^{+\frac{b}{2}} \left[\exp \left(\frac{i\pi y}{b} \right) + \exp \left(-\frac{i\pi y}{b} \right) \right] \exp \left[-\frac{ik}{2} \left(\frac{y^2}{l_H} \right) \right] \exp \{ ik(\beta y) \} dy = \\ & \int_{-\frac{b}{2}}^{+\frac{b}{2}} \exp \left\{ -i \left[\frac{ky^2}{2l_H} - y \left(k\beta + \frac{\pi}{b} \right) \right] \right\} dy + \int_{-\frac{b}{2}}^{+\frac{b}{2}} \exp \left\{ -i \left[\frac{ky^2}{2l_H} - y \left(k\beta - \frac{\pi}{b} \right) \right] \right\} dy \end{aligned} \quad (12.1.9)$$

Osserviamo che:

$$\frac{ky^2}{2l_H} - y \left(k\beta \pm \frac{\pi}{b} \right) = \frac{k}{2l_H} \left[y^2 - y \frac{2l_H}{k} \left(k\beta \pm \frac{\pi}{b} \right) \right] \quad (12.1.10)$$

Dentro l'espressione fra parentesi quadra della (12.1.10) aggiungiamo e sottraiamo la quantità $\left[\frac{l_H}{k} \left(k\beta \pm \frac{\pi}{b} \right) \right]^2$; si ha:

$$\begin{aligned} \frac{ky^2}{2l_H} - y \left(k\beta \pm \frac{\pi}{b} \right) &= \frac{k}{2l_H} \left\{ y^2 - y \frac{2l_H}{k} \left(k\beta \pm \frac{\pi}{b} \right) + \left[\frac{l_H}{k} \left(k\beta \pm \frac{\pi}{b} \right) \right]^2 \right\} - \frac{k}{2l_H} \left[\frac{l_H}{k} \left(k\beta \pm \frac{\pi}{b} \right) \right]^2 \\ &= \frac{k}{2l_H} \left[y - \frac{l_H}{k} \left(k\beta \pm \frac{\pi}{b} \right) \right]^2 - \frac{l_H}{2k} \left(k\beta \pm \frac{\pi}{b} \right)^2 \end{aligned}$$

La (12.1.9), pertanto, è uguale a:

$$\begin{aligned} & \exp \left[i \frac{l_H}{2k} \left(k\beta + \frac{\pi}{b} \right)^2 \right] \int_{-\frac{b}{2}}^{+\frac{b}{2}} \exp \left\{ -i \frac{k}{2l_H} \left[y - \frac{l_H}{k} \left(k\beta + \frac{\pi}{b} \right) \right]^2 \right\} dy + \\ & + \exp \left[i \frac{l_H}{2k} \left(k\beta - \frac{\pi}{b} \right)^2 \right] \int_{-\frac{b}{2}}^{+\frac{b}{2}} \exp \left\{ -i \frac{k}{2l_H} \left[y - \frac{l_H}{k} \left(k\beta - \frac{\pi}{b} \right) \right]^2 \right\} dy \end{aligned} \quad (12.1.11)$$

che si può trasformare in integrali di Fresnel ponendo $v = \sqrt{\frac{2}{\lambda l_H}} \left[y - \frac{\lambda l_H}{2\pi} \left(\frac{2\pi}{\lambda} \beta + \frac{\pi}{b} \right) \right] \Rightarrow$
 $dv = \sqrt{\frac{2}{\lambda l_H}} dy$ nel primo integrale e $w = \sqrt{\frac{2}{\lambda l_H}} \left[y - \frac{\lambda l_H}{2\pi} \left(\frac{2\pi}{\lambda} \beta - \frac{\pi}{b} \right) \right] \Rightarrow dw = \sqrt{\frac{2}{\lambda l_H}} dy$
 nel secondo integrale.

Finalmente la (12.1.9) si può scrivere:

$$\begin{aligned} & \sqrt{\frac{\lambda l_H}{2}} \exp \left[i \frac{l_H \lambda}{4\pi} \left(\frac{2\pi}{\lambda} \beta + \frac{\pi}{b} \right)^2 \right] \int_{v_1}^{v_2} \exp \left\{ -i \frac{\pi}{2} v^2 \right\} dv + \\ & + \sqrt{\frac{\lambda l_H}{2}} \exp \left[i \frac{l_H \lambda}{4\pi} \left(\frac{2\pi}{\lambda} \beta - \frac{\pi}{b} \right)^2 \right] \int_{w_1}^{w_2} \exp \left\{ -i \frac{\pi}{2} w^2 \right\} dw \end{aligned} \quad (12.1.12)$$

essendo:

$$\begin{aligned} v_1 &= - \left\{ \sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} + \frac{2\beta}{\lambda} \sqrt{\frac{\lambda l_H}{2}} + \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\} \\ v_2 &= \left\{ \sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} - \frac{2\beta}{\lambda} \sqrt{\frac{\lambda l_H}{2}} - \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\} \\ w_1 &= \left\{ -\sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} - \frac{2\beta}{\lambda} \sqrt{\frac{\lambda l_H}{2}} + \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\} \\ w_2 &= \left\{ \sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} - \frac{2\beta}{\lambda} \sqrt{\frac{\lambda l_H}{2}} + \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\} \end{aligned} \quad (12.1.13)$$

La (12.1.12) si può anche scrivere:

$$\begin{aligned} & \sqrt{\frac{\lambda l_H}{2}} \exp \left[i \frac{l_H \lambda}{4\pi} \left(\frac{2\pi}{\lambda} \beta + \frac{\pi}{b} \right)^2 \right] \{ \mathcal{F}(v_2) + \mathcal{F}(-v_1) \} \\ & + \sqrt{\frac{\lambda l_H}{2}} \exp \left[i \frac{l_H \lambda}{4\pi} \left(\frac{2\pi}{\lambda} \beta - \frac{\pi}{b} \right)^2 \right] \{ \mathcal{F}(w_2) + \mathcal{F}(-w_1) \} \end{aligned} \quad (12.1.14)$$

In definitiva

$$\begin{aligned} F_x(\alpha, \beta) &= \frac{E_0}{2\lambda^2} \sqrt{\frac{\lambda l_E}{2}} \exp \left(i \frac{\pi \alpha^2 l_E}{\lambda} \right) (\mathcal{F}(u_2) + \mathcal{F}(-u_1)) \cdot \\ & \cdot \left\{ \sqrt{\frac{\lambda l_H}{2}} \exp \left[i \frac{l_H \lambda}{4\pi} \left(\frac{2\pi}{\lambda} \beta + \frac{\pi}{b} \right)^2 \right] [\mathcal{F}(v_2) + \mathcal{F}(-v_1)] + \right. \\ & \left. + \sqrt{\frac{\lambda l_H}{2}} \exp \left[i \frac{l_H \lambda}{4\pi} \left(\frac{2\pi}{\lambda} \beta - \frac{\pi}{b} \right)^2 \right] [\mathcal{F}(w_2) + \mathcal{F}(-w_1)] \right\} \end{aligned} \quad (12.1.15)$$

Il campo “far field” irradiato dalla struttura piramidale è dato dalle formule (8.13.8), (8.13.9).

Esse, come abbiamo visto, si possono applicare purchè sia soddisfatto il criterio di Rayleigh $r \geq \frac{2D^2}{\lambda}$.

Si ha, pertanto:

$$\begin{aligned}\vec{E}(r, \theta, \phi) &\simeq i2\pi \frac{\exp(-ikr)}{kr} (\hat{e}_\theta \cos \phi - \hat{e}_\phi \cos \theta \sin \phi) F_x(\alpha, \beta) \\ \vec{H}(r, \theta, \phi) &\simeq \frac{1}{Z} \hat{e}_r \times \vec{E}(r, \theta, \phi)\end{aligned}\tag{12.1.16}$$

con $\alpha = \sin \theta \cos \phi$ e $\beta = \sin \theta \sin \phi$.

Vogliamo studiare il diagramma di radiazione nel piano E cioè nel piano principale xz ($\phi = 0, \beta = 0$).

Si ha allora:

$$E_\theta \simeq i2\pi \frac{\exp(-ikr)}{kr} F_x(\alpha, 0)\tag{12.1.17}$$

Calcoliamo $F_x(\alpha, 0)$:

$$\begin{aligned}F_x(\alpha, 0) &= \frac{E_0}{2\lambda^2} \sqrt{\frac{\lambda l_E}{2}} \exp\left(i \frac{\pi (\sin^2 \theta) l_E}{\lambda}\right) (\mathcal{F}(u_2) + \mathcal{F}(-u_1)) \cdot \\ &\cdot \left\{ \sqrt{\frac{\lambda l_H}{2}} \exp\left[i \frac{l_H \lambda \pi}{4b^2}\right] [\mathcal{F}(v'_2) + \mathcal{F}(-v'_1)] + \right. \\ &\left. + \sqrt{\frac{\lambda l_H}{2}} \exp\left[i \frac{l_H \lambda \pi}{4b^2}\right] [\mathcal{F}(w'_2) + \mathcal{F}(-w'_1)] \right\}\end{aligned}\tag{12.1.18}$$

essendo

$$\begin{aligned}v'_1 &= - \left\{ \sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} + \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\} \\ v'_2 &= \left\{ \sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} - \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\} \\ w'_1 &= \left\{ -\sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} + \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\} \\ w'_2 &= \left\{ \sqrt{\frac{2}{\lambda l_H}} \frac{b}{2} + \frac{1}{b} \sqrt{\frac{\lambda l_H}{2}} \right\}\end{aligned}\tag{12.1.19}$$

Vogliamo graficare il diagramma di radiazione e pertanto calcoliamo:

$$\langle \vec{S} \rangle = \frac{1}{2} \Re (\vec{E} \times \vec{H}^*)$$

che per la (12.1.16) si scrive:

$$\langle \vec{S} \rangle = \frac{1}{2Z} \Re (E_\theta E_\theta^* \hat{e}_r) = \frac{1}{2Z} |E_\theta|^2 \hat{e}_r\tag{12.1.20}$$

Pertanto:

$$\langle \vec{S} \rangle = \frac{1}{2Z} \frac{4\pi^2}{k^2 r^2} |F_x(\alpha, 0)|^2 \hat{e}_r \quad (12.1.21)$$

essendo:

$$|F_x(\alpha, 0)|^2 = \frac{E_0^2}{4\lambda^4} \frac{\lambda l_E}{2} |\mathcal{F}(u_2) + \mathcal{F}(-u_1)|^2.$$

$$\cdot \left\{ \left[\sqrt{\frac{\lambda l_H}{2}} \exp\left(i \frac{l_H \lambda \pi}{4b^2}\right) [\mathcal{F}(v'_2) + \mathcal{F}(-v'_1)] + \sqrt{\frac{\lambda l_H}{2}} \exp\left(i \frac{l_H \lambda \pi}{4b^2}\right) [\mathcal{F}(w'_2) + \mathcal{F}(-w'_1)] \right] \right\}^2 \quad (12.1.22)$$

Osserviamo che l'unica dipendenza da θ è contenuta nel termine $|\mathcal{F}(u_2) + \mathcal{F}(-u_1)|^2$; tutti gli altri termini, una volta fissata la geometria del sistema e la lunghezza d'onda della radiazione emessa, sono costanti.

Consideriamo le (12.1.7)

$$\begin{aligned} u_2 &= \left\{ \sqrt{\frac{2}{\lambda l_E}} \frac{a}{2} - \frac{2\alpha}{\lambda} \sqrt{\frac{\lambda l_E}{2}} \right\} \\ u_1 &= - \left\{ \sqrt{\frac{2}{\lambda l_E}} \frac{a}{2} + \frac{2\alpha}{\lambda} \sqrt{\frac{\lambda l_E}{2}} \right\} \end{aligned} \quad (12.1.7)$$

Per $-\frac{\pi}{2} \leq \theta < 0$ α diventa negativo, si ha allora:

$$u_{2(\theta < 0)} = -u_{1(\theta > 0)} \quad ; \quad u_{1(\theta < 0)} = -u_{2(\theta > 0)} \quad (12.1.23)$$

Segue quindi che:

$$|\mathcal{F}(u_{2(\theta < 0)}) + \mathcal{F}(-u_{1(\theta < 0)})|^2 = |\mathcal{F}(-u_{1(\theta > 0)}) + \mathcal{F}(u_{2(\theta > 0)})|^2$$

cioè il diagramma di radiazione è simmetrico rispetto alla direzione $\theta = 0^0$ in corrispondenza della quale, per evidenti ragioni fisiche, presenta un massimo assoluto.

È conveniente graficare, al solito, la funzione normalizzata

$$K(\theta) = \frac{\langle \vec{S} \rangle}{|\langle \vec{S} \rangle|_{\max}} = \frac{|\mathcal{F}(u_2) + \mathcal{F}(-u_1)|^2}{|\mathcal{F}(u_2) + \mathcal{F}(-u_1)|_{\max}^2} \quad (12.1.24)$$

che si può ancora scrivere:

$$\begin{aligned} K(\theta) &= \frac{|\mathcal{C}(u_2) - i\mathcal{S}(u_2) - \mathcal{C}(u_1) + i\mathcal{S}(u_1)|^2}{|\mathcal{C}(u_2) - i\mathcal{S}(u_2) - \mathcal{C}(u_1) + i\mathcal{S}(u_1)|_{\max}^2} = \\ &= \frac{[\mathcal{C}(u_2) - \mathcal{C}(u_1)]^2 + [\mathcal{S}(u_2) - \mathcal{S}(u_1)]^2}{\left\{ [\mathcal{C}(u_2) - \mathcal{C}(u_1)]^2 + [\mathcal{S}(u_2) - \mathcal{S}(u_1)]^2 \right\}_{\max}} \end{aligned} \quad (12.1.25)$$

essendo (come vedremo nei calcoli) il massimo in corrispondenza di $\theta = 0^0$, si ha che:

$$u_{2(\theta=0)} = -u_{1(\theta=0)} = \sqrt{\frac{2}{\lambda l_E}} \frac{a}{2} = u^* \quad (12.1.26)$$

Pertanto:

$$\left\{ [\mathcal{C}(u_2) - \mathcal{C}(u_1)]^2 + [\mathcal{S}(u_2) - \mathcal{S}(u_1)]^2 \right\}_{\max} = 4 [\mathcal{C}(u^*)]^2 + 4 [\mathcal{S}(u^*)]^2 \quad (12.1.27)$$

La (12.1.25) diventa allora:

$$K(\theta) = \frac{[\mathcal{C}(u_2) - \mathcal{C}(u_1)]^2 + [\mathcal{S}(u_2) - \mathcal{S}(u_1)]^2}{4 [\mathcal{C}(u^*)]^2 + 4 [\mathcal{S}(u^*)]^2} \quad (12.1.28)$$

GRAFICO I

$$a = 2.40\lambda \quad l_E = 4.21\lambda$$

(esempio: $\lambda = 3\text{cm}$ ($\nu = 10\text{GHz}$), $a = 7.2\text{cm}$, $l_E = 12.63\text{cm}$)

$$u_2 = \sqrt{\frac{2}{4.21}} \frac{2.40}{2} - 2 \sin \theta \sqrt{\frac{4.21}{2}} = 0.8271 - 2.90172 \sin \theta$$

$$u_1 = - \left[\sqrt{\frac{2}{4.21}} \frac{2.40}{2} + 2 \sin \theta \sqrt{\frac{4.21}{2}} \right] = -0.8271 - 2.90172 \sin \theta$$

$$A = [\mathcal{C}(u_2) - \mathcal{C}(u_1)]^2, \quad B = [\mathcal{S}(u_2) - \mathcal{S}(u_1)]^2$$

GRAFICO II

$$a = 4.50\lambda \quad l_E = 10\lambda$$

$$u_2 = \sqrt{\frac{2}{10}} \frac{4.5}{2} - 2 \sin \theta \sqrt{\frac{10}{2}} = 1.00623059 - 4.472125955 \sin \theta$$

$$u_1 = - \left[\sqrt{\frac{2}{10}} \frac{4.5}{2} + 2 \sin \theta \sqrt{\frac{10}{2}} \right] = -1.00623059 - 4.472135955 \sin \theta$$

$$A = [\mathcal{C}(u_2) - \mathcal{C}(u_1)]^2, \quad B = [\mathcal{S}(u_2) - \mathcal{S}(u_1)]^2$$

I valori di \mathcal{C} e \mathcal{S} per $u > 5$ sono dati dalla formula associata alla tabella di pagina 322 dell'Abramowitz e riportata nel paragrafo F7 del Formulario.

Tabella I

θ	u_2	u_1	$\mathcal{C}(u_2)$	$\mathcal{C}(u_1)$	$\mathcal{S}(u_2)$	$\mathcal{S}(u_1)$	A	B	$(A+B)$
-90^0	3.7288	2.0746	0.51471	0.5601	0.58398	0.3608	0.0020604	0.049809	0.051869
-85^0	3.7178	2.0636	0.52549	0.55012	0.58162	0.35608	$6.07 \cdot 10^{-4}$	0.050869	0.051476
-80^0	3.6847	2.0305	0.55474	0.51861	0.56668	0.34635	0.0013051	0.048545	0.04985
-75^0	3.6299	1.9758	0.58477	0.4641	0.52194	0.34525	0.014561	0.031219	0.04578
-70^0	3.5538	1.8996	0.57347	0.39417	0.44901	0.37356	0.032147	0.0056926	0.03784
-65^0	3.4569	1.8028	0.49044	0.33467	0.40858	0.44838	0.024265	0.0015841	0.025849
-60^0	3.3401	1.6859	0.40711	0.32678	0.47954	0.56301	0.0064541	0.0069669	0.013421
-55^0	3.204	1.5499	0.46259	0.40189	0.59178	0.67316	0.0036847	0.006622	0.010307
-50^0	3.4899	1.3958	0.59412	0.54733	0.54441	0.71322	0.0021894	0.028496	0.030686
-45^0	2.8789	1.2247	0.54442	0.6988	0.39917	0.64169	0.023835	0.058815	0.08265
-40^0	2.6923	1.0381	0.38933	0.77759	0.45995	0.47626	0.15074	$2.66 \cdot 10^{-4}$	0.15101
-35^0	2.4915	0.83727	0.46541	0.74129	0.62218	0.28171	0.076107	0.11593	0.19203
-30^0	2.278	0.62377	0.63465	0.60087	0.53267	0.12373	0.0011413	0.16724	0.16838
-25^0	2.0534	0.39923	0.54065	0.39673	0.35237	0.033168	0.020713	0.10189	0.12261
-20^0	1.8195	0.16535	0.34182	0.16532	0.4332	0.0023669	0.031151	0.18562	0.21677
-15^0	1.5781	-0.076073	0.38031	-0.076073	0.65495	$-2.31 \cdot 10^{-4}$	0.20828	0.42927	0.63755
-10^0	1.331	-0.32322	0.61033	-0.32235	0.69904	-0.017646	0.86988	0.51365	1.3835
-5^0	1.08	-0.57419	0.76963	-0.55898	0.51737	-0.09724	1.7652	0.37774	2.143
0^0	0.82709	-0.82709	0.73656	-0.73656	0.2727	-0.2727	2.1701	0.29746	2.4676

Tabella II

θ	u_2	u_1	$\mathcal{C}(u_2)$	$\mathcal{C}(u_1)$	$\mathcal{S}(u_2)$	$\mathcal{S}(u_1)$	A	B	$(A+B)$
-90^0	5.4784	3.4659	0.49947	0.49939	0.55808	0.40832	$7.02 \cdot 10^{-9}$	0.02243	0.02243
-85^0	5.4613	3.4489	0.5163	0.48244	0.55594	0.40956	0.0011461	0.021429	0.022575
-80^0	5.4104	3.3978	0.55376	0.43694	0.52385	0.43096	0.013649	0.0086273	0.022276
-75^0	5.326	3.3135	0.53193	0.40431	0.4495	0.50585	0.016287	0.0031748	0.019462
-70^0	5.2087	3.1962	0.44004	0.46988	0.4883	0.59468	$8.90 \cdot 10^{-4}$	0.011317	0.012208
-65^0	5.0594	3.0469	0.53782	0.59547	0.55025	0.54168	0.0033237	$7.33 \cdot 10^{-5}$	0.003397
-60^0	4.8792	2.8668	0.47968	0.5332	0.43804	0.39446	0.0028648	0.0018991	0.0047639
-55^0	4.6696	2.6571	0.52148	0.38093	0.56466	0.49396	0.019755	0.0049981	0.024753
-50^0	4.4321	2.4196	0.46089	0.53625	0.43982	0.62554	0.0056791	0.03449	0.040169
-45^0	4.1685	2.156	0.56411	0.61915	0.54137	0.41541	0.0030292	0.015866	0.018895
-40^0	3.8809	1.8684	0.4183	0.37047	0.49386	0.39383	0.0022878	0.010007	0.012295
-35^0	3.5713	1.5589	0.5816	0.39472	0.46449	0.66767	0.034924	0.041283	0.076206
-30^0	3.2423	1.2298	0.43145	0.69516	0.56998	0.64526	0.06954	0.0056682	0.075208
-25^0	2.8962	0.88378	0.55933	0.7597	0.40793	0.32438	0.040149	0.0069806	0.047129
-20^0	2.5358	0.52333	0.42671	0.51373	0.60103	0.074059	0.0075727	0.2777	0.28527
-15^0	2.1637	0.15124	0.62299	0.15122	0.42203	0.0018113	0.22256	0.17659	0.39915
-10^0	1.7828	-0.22965	0.32809	-0.22949	0.4672	-0.0063387	0.3109	0.22424	0.53514
-5^0	1.396	-0.61646	0.54708	-0.59485	0.71324	-0.11958	1.1304	0.69358	1.9976
0^0	1.0062	-1.0062	0.77983	-0.77983	0.44449	-0.44449	2.4326	0.79028	3.2228

GRAFICO I

Riportiamo i valori dell'ultima colonna della tabella relativa al grafico 1 divisi per il valore massimo della stessa colonna corrispondente a $\theta = 0$; riportiamo, cioè, la funzione $K(\theta)$.

θ	$K(\theta)$	$10 \log_{10} K(\theta)$ [dB]	θ	$K(\theta)$	$10 \log_{10} K(\theta)$ [dB]
-90 ⁰	0.02102	-16.774	-40 ⁰	0.061198	-12.133
-85 ⁰	0.020861	-16.807	-35 ⁰	0.077823	-11.089
-80 ⁰	0.020202	-16.946	-30 ⁰	0.068236	-11.66
-75 ⁰	0.018553	-17.316	-25 ⁰	0.049687	-13.038
-70 ⁰	0.015335	-18.143	-20 ⁰	0.087849	-10.563
-65 ⁰	0.010476	-19.798	-15 ⁰	0.25837	-5.8776
-60 ⁰	0.005439	-22.645	-10 ⁰	0.56069	-2.5128
-55 ⁰	0.0041769	-23.791	-5 ⁰	0.86845	-0.61255
-50 ⁰	0.012436	-19.053	0 ⁰	1	0
-45 ⁰	0.033495	-14.75	◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇		

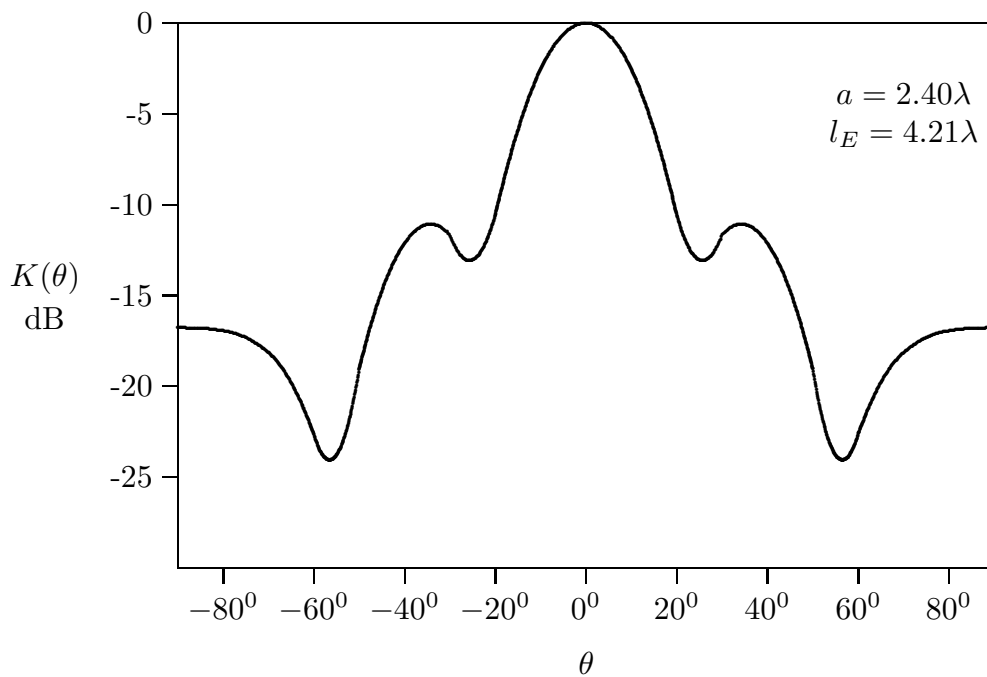


fig.12.1.2

GRAFICO II

Riportiamo i valori dell'ultima colonna della tabella relativa al grafico 2 divisi per il valore massimo della stessa colonna corrispondente a $\theta = 0$; riportiamo, cioè, la funzione $K(\theta)$.

θ	$K(\theta)$	$10 \log_{10} K(\theta)$ [dB]	θ	$K(\theta)$	$10 \log_{10} K(\theta)$ [dB]
-90^0	$6.9596 \cdot 10^{-3}$	-21.574	-40^0	$3.815 \cdot 10^{-3}$	-24.185
-85^0	$7.0047 \cdot 10^{-3}$	-21.546	-35^0	$2.3646 \cdot 10^{-2}$	-16.262
-80^0	$6.9119 \cdot 10^{-3}$	-21.604	-30^0	$2.3336 \cdot 10^{-2}$	-16.32
-75^0	$6.0387 \cdot 10^{-3}$	-22.191	-25^0	$1.4624 \cdot 10^{-2}$	-18.349
-70^0	$3.7878 \cdot 10^{-3}$	-24.216	-20^0	$8.8515 \cdot 10^{-2}$	-10.53
-65^0	$1.0540 \cdot 10^{-3}$	-29.771	-15^0	$1.2385 \cdot 10^{-1}$	-9.071
-60^0	$1.4782 \cdot 10^{-3}$	-28.303	-10^0	$1.6605 \cdot 10^{-1}$	-7.7977
-55^0	$7.6804 \cdot 10^{-3}$	-21.146	-5^0	$6.1982 \cdot 10^{-1}$	-2.0773
-50^0	$1.2464 \cdot 10^{-2}$	-19.043	0^0	1	0
-45^0	$5.8629 \cdot 10^{-3}$	-22.319	◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇ ◇		

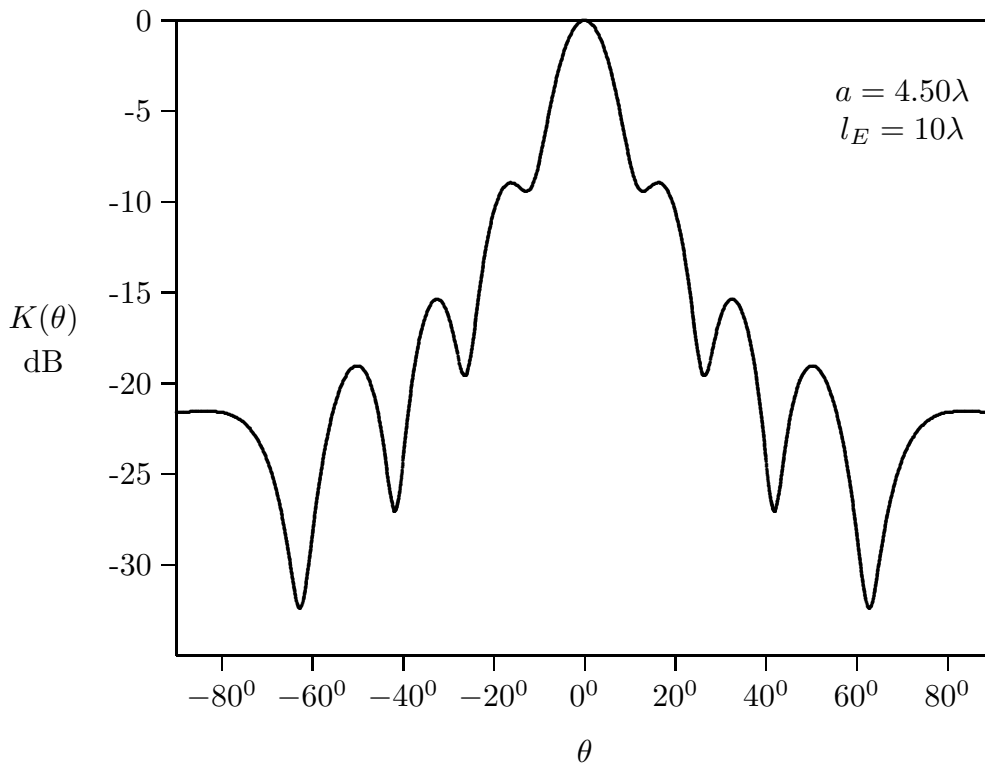


fig.12.1.3

Diagrammi di radiazione 3D
Antenna Piramidale: Diffrazione di Fraunhofer ($z=100\lambda$)

$\nu=10$ GHz, $\lambda=3$ cm, $a=1\lambda$, $b=a$, $I_E=I_H=10\lambda$, $z=100\lambda$

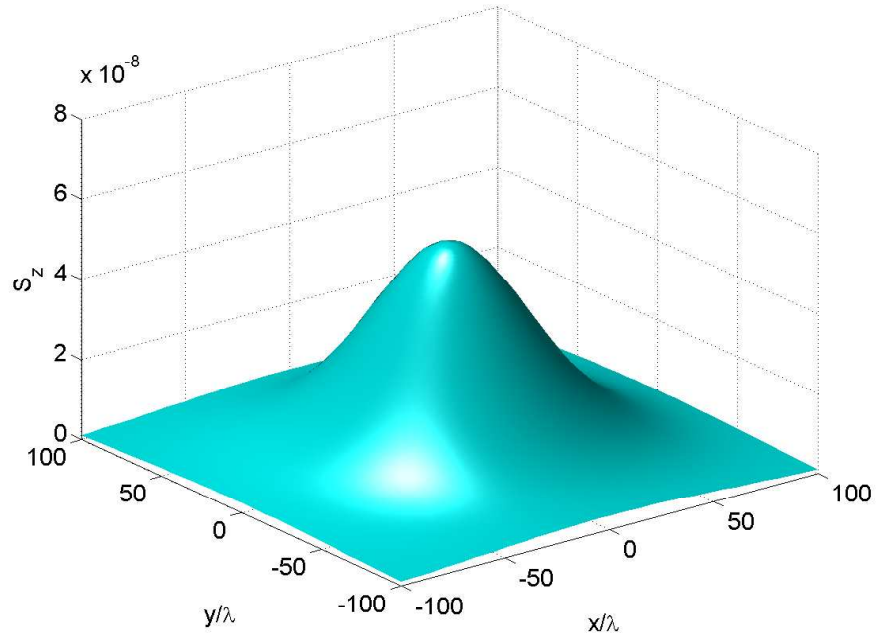


fig.12.1-1

$\nu=10$ GHz, $\lambda=3$ cm, $a=2\lambda$, $b=a$, $I_E=I_H=10\lambda$, $z=100\lambda$

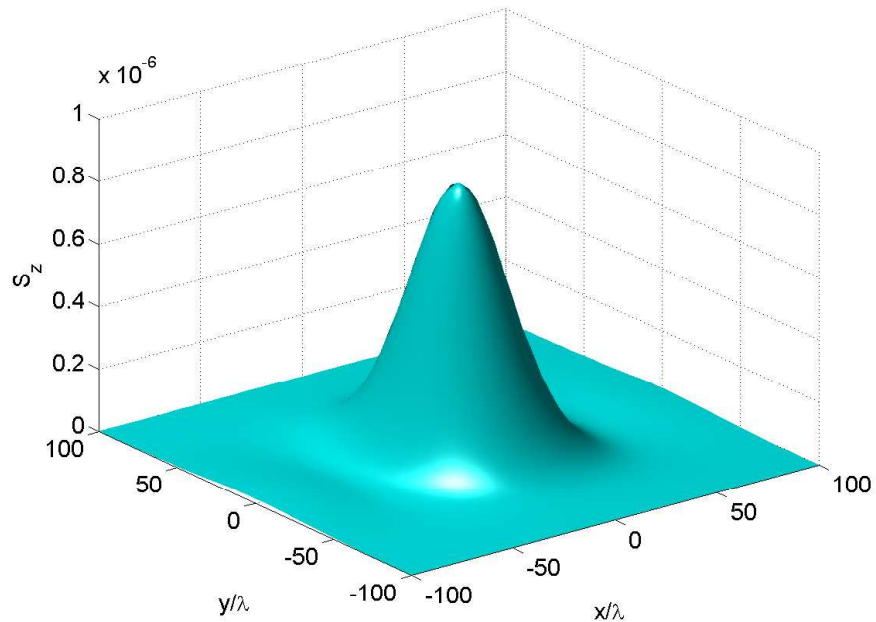


fig.12.1-2

$\nu=10$ GHz, $\lambda=3$ cm, $a=3\lambda$, $b=a$, $I_E=I_H=10\lambda$, $z=100\lambda$

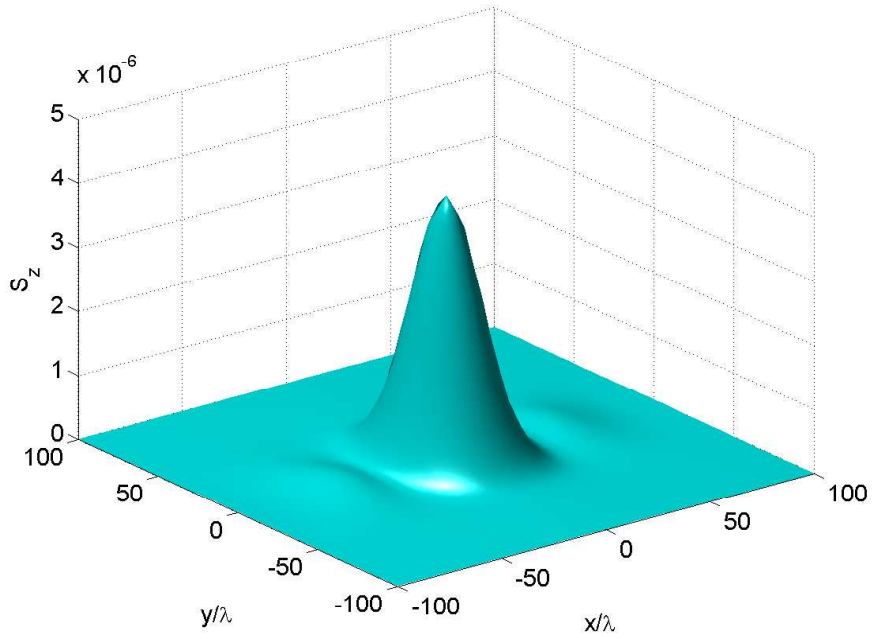


fig.12.1-3

$\nu=10$ GHz, $\lambda=3$ cm, $a=4\lambda$, $b=a$, $I_E=I_H=10\lambda$, $z=100\lambda$

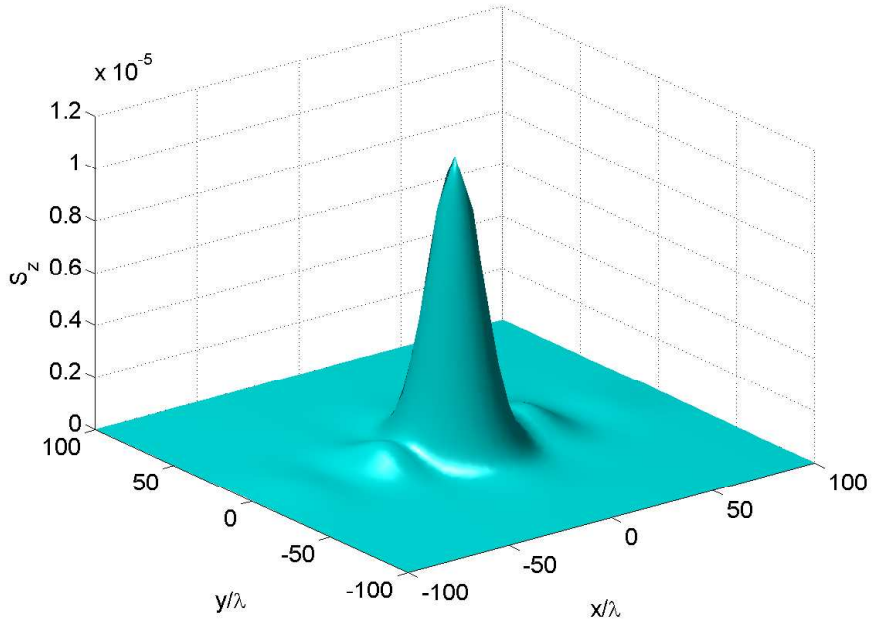


fig.12.1-4

$\nu=10$ GHz, $\lambda=3$ cm, $a=5\lambda$, $b=a$, $I_E=I_H=10\lambda$, $z=100\lambda$

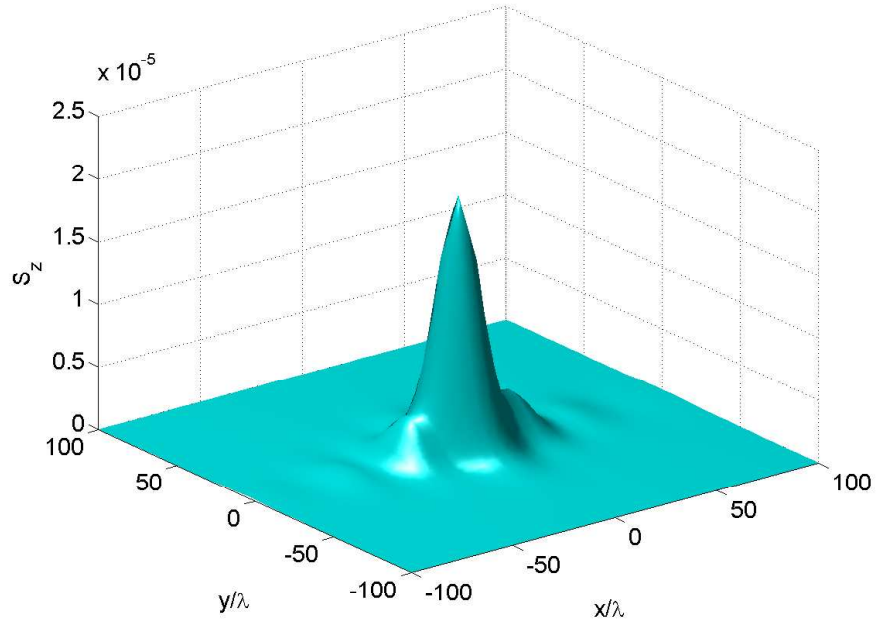


fig.12.1-5

$\nu=10$ GHz, $\lambda=3$ cm, $a=6\lambda$, $b=a$, $I_E=I_H=10\lambda$, $z=100\lambda$

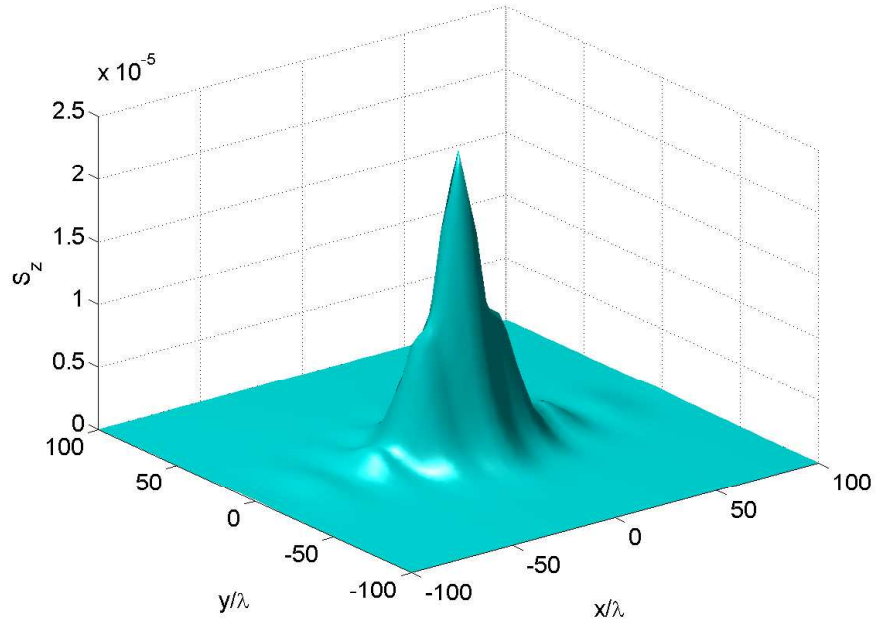


fig.12.1-6

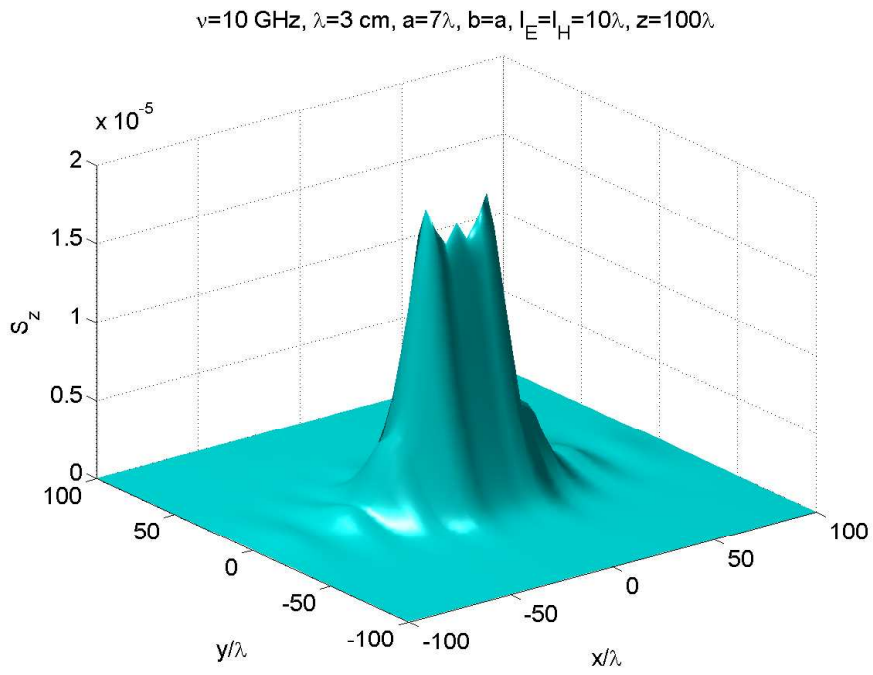


fig.12.1-7

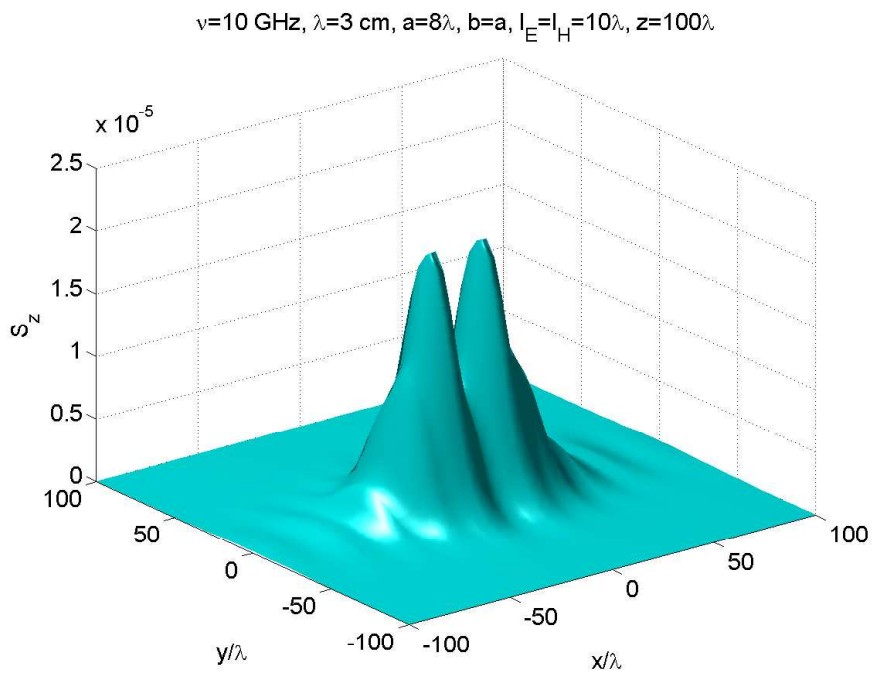


fig.12.1-8

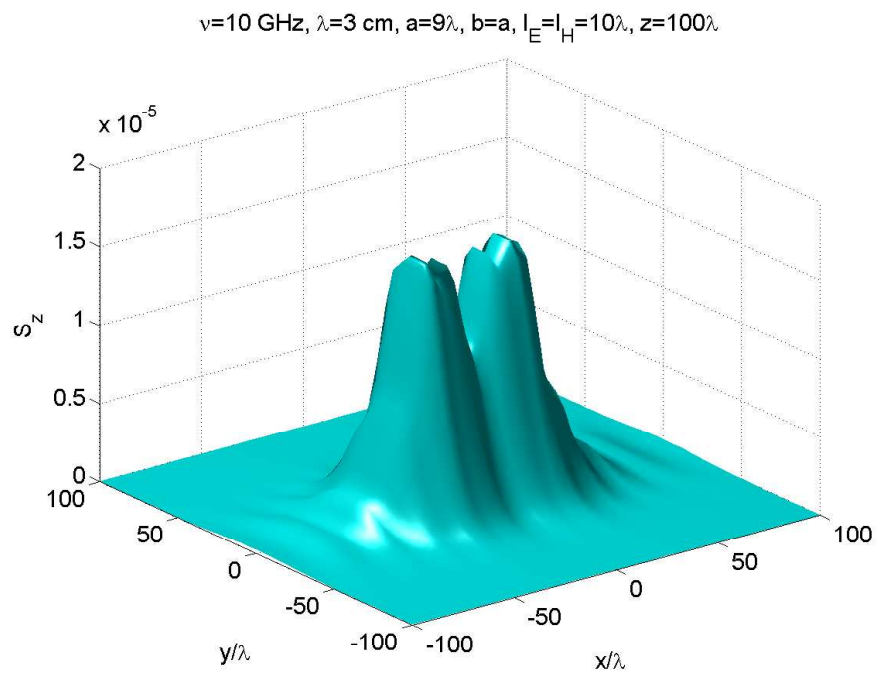


fig.12.1-9

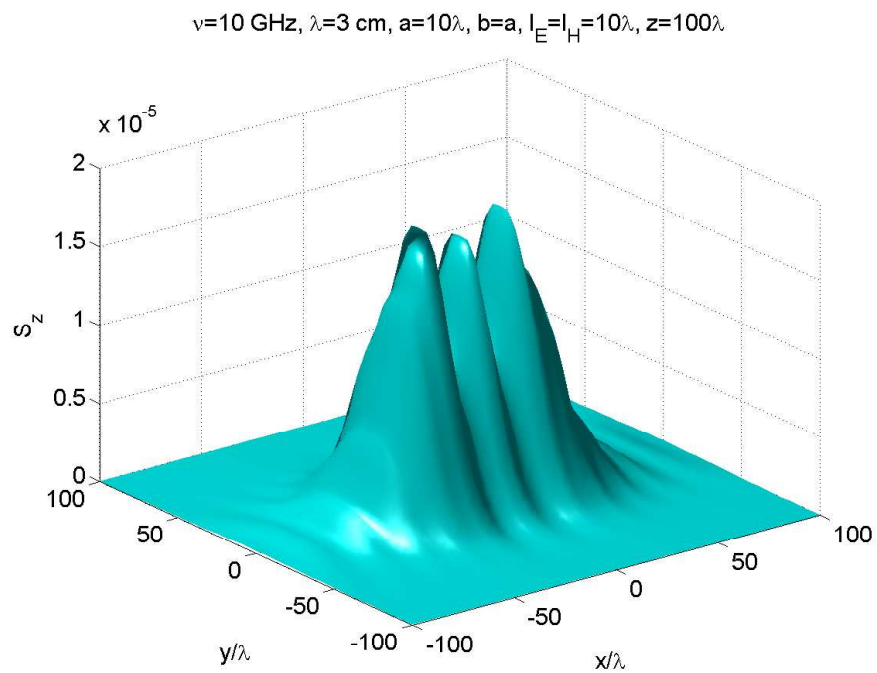


fig.12.1-10

Horn Piramidale: Diffrazione di Fraunhofer Programma 3D in ambiente Matlab

```
01) - %%%%% Diffrazione di Fraunhofer da horn piramidale %%%%%
02) - delete(get(0,'children'));
03) - clear all;
04) - freq=10e9; %%%%% frequenza della radiazione emessa %%%%%
05) - c=3e8;
06) - lambda=c./freq; %%%%% lunghezza d'onda %%%%%
07) - a=1.*lambda; %%%%% lato a dell'horn piramidale %%%%%
08) - b=a; %%%%% lato b dell'horn piramidale %%%%%
09) - E0=1;
10) - Z=377; %%%%% Impedenza del vuoto %%%%%
11) - k=2.*pi./lambda;
12) - lE=10.*lambda;
13) - lH=lE;
14) - [x,y] = meshgrid([-100.*lambda:5.*lambda:+100.*lambda]);
15) - z=100.*lambda;
16) - r=sqrt(x.^2+y.^2+z.^2);
17) - alfa=x./r;
18) - beta=y./r;
19) - gamma=z./r;
20) - u1=-(sqrt(2./lambda./lE).*a./2+(2.*alfa./lambda).*sqrt(lambda.*lE./2));
21) - u2=+(sqrt(2./lambda./lE).*a./2-(2.*alfa./lambda).*sqrt(lambda.*lE./2));
22) - v1=-(sqrt(2./lambda./lH).*b./2+(2.*beta./lambda+1./b).*
sqrt(lambda.*lH./2));
23) - v2=+(sqrt(2./lambda./lH).*b./2-(2.*beta./lambda+1./b).*
sqrt(lambda.*lH./2));
24) - w1=-(sqrt(2./lambda./lH).*b./2+(2.*beta./lambda-1./b).*
sqrt(lambda.*lH./2));
25) - w2=+(sqrt(2./lambda./lH).*b./2-(2.*beta./lambda-1./b).*
sqrt(lambda.*lH./2));
26) - Cu1=mfun('FresnelC',u1);
27) - Su1=mfun('FresnelS',u1);
28) - Cu2=mfun('FresnelC',u2);
29) - Su2=mfun('FresnelS',u2);
30) - Cv1=mfun('FresnelC',v1);
31) - Sv1=mfun('FresnelS',v1);
32) - Cv2=mfun('FresnelC',v2);
33) - Sv2=mfun('FresnelS',v2);
34) - Cw1=mfun('FresnelC',w1);
35) - Sw1=mfun('FresnelS',w1);
36) - Cw2=mfun('FresnelC',w2);
37) - Sw2=mfun('FresnelS',w2);
```



```

38) - Fu2=Cu2-i.*Su2;
39) - Fu1=Cu1-i.*Su1;
40) - Fv2=Cv2-i.*Sv2;
41) - Fv1=Cv1-i.*Sv1;
42) - Fw2=Cw2-i.*Sw2;
43) - Fw1=Cw1-i.*Sw1;
44) - %xxxxx Funzione F_x xxxxxxxx
45) - A1=E0./(2.*lambda.^2).*sqrt(lambda.*lE./2).*
exp(i.*(pi.*alfa.^2.*lE./lambda)).*(Fu2-Fu1);
46) - A2=sqrt(lambda.*lH./2).*exp(i.*(lH.*lambda./4./pi).*
(2.*pi./lambda.*beta+pi./b).^2).*(Fv2-Fv1);
47) - A3=sqrt(lambda.*lH./2).*exp(i.*(lH.*lambda./4./pi).*
(2.*pi./lambda.*beta-pi./b).^2).*(Fw2-Fw1);
48) - Fx=A1.*(A2+A3);
49) - S=1./2./Z.*(2.*pi).^2./(k.*r).^2.*
(1-beta.^2).*abs(Fx).^2;
50) - Sz=S.*gamma;
51) - Smax=max(Sz);
52) - surf(x./lambda, y./lambda, Sz, 'FaceColor', 'cyan', 'EdgeColor', 'None')
53) - camlight left; lighting phong
54) - set(gca,'fontsize',15);
55) - xlabel('x/λ');
56) - ylabel('y/λ');
57) - zlabel('S_z');
58) - title('ν=10 GHz, λ=3 cm, a=1λ, b=a, lE=lH=10λ, z = 100λ')
59) - %view([60 90])
60) - print('-depsc2','hornpiramidalefraunhofer_1')

```

12.2 - Diffrazione di Fresnel per l'antenna ad horn rettangolare

Consideriamo solo la componente lungo l'asse x , perché quella lungo l'asse y non esiste in quanto nell'horn rettangolare il campo é solo lungo x .

$$\vec{E}(x, y, z) = \frac{i}{\lambda z} \exp(-ikz) \hat{x} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_{0x} \exp \left[-ik \frac{(x - \xi)^2 + (y - \eta)^2}{2z} \right] d\xi d\eta \quad (12.2.1)$$

$$E_{0x} = E_{0rect_{a,b}}(\xi, \eta) \cos \left(\frac{\pi\eta}{b} \right) \exp \left[-\frac{ik}{2} \left(\frac{\xi^2}{l_E} + \frac{\eta^2}{l_H} \right) \right] \quad (12.2.2)$$

Pertanto l'integrale (12.2.1) si scrive:

$$\begin{aligned} & \int_{-a/2}^{+a/2} \int_{-b/2}^{+b/2} \cos \left(\frac{\pi\eta}{b} \right) \exp \left[-\frac{ik}{2} \left(\frac{\xi^2}{l_E} \right) \right] \exp \left[-\frac{ik}{2} \left(\frac{\eta^2}{l_H} \right) \right] \cdot \\ & \cdot \exp \left[-\frac{ik}{2z} (x - \xi)^2 \right] \exp \left[-\frac{ik}{2z} (x - \eta)^2 \right] d\xi d\eta = \\ & = \left\{ \int_{-a/2}^{+a/2} \exp \left[-\frac{ik}{2} \left(\frac{\xi^2}{l_E} \right) \right] \cdot \exp \left[-\frac{ik}{2z} (x^2 - 2x\xi + \xi^2) \right] d\xi \right\} \cdot \\ & \cdot \left\{ \int_{-b/2}^{+b/2} \cos \left(\frac{\pi\eta}{b} \right) \exp \left[-\frac{ik}{2} \left(\frac{\eta^2}{l_H} \right) \right] \exp \left[-\frac{ik}{2z} (y^2 - 2y\eta + \eta^2) \right] d\eta \right\} \end{aligned} \quad (12.2.3)$$

Il primo integrale della (12.2.3) si può scrivere:

$$\begin{aligned} & \int_{-a/2}^{+a/2} \exp \left[-\frac{ik}{2} \left(\frac{\xi^2}{l_E} \right) \right] \cdot \exp \left[-\frac{ik}{2z} (x^2 - 2x\xi + \xi^2) \right] d\xi = \\ & = \exp \left(-\frac{ik}{2z} x^2 \right) \int_{-a/2}^{+a/2} \exp \left[-\frac{ik}{2} \left(\frac{\xi^2}{l_E} - \frac{2x\xi}{z} + \frac{\xi^2}{z} \right) \right] d\xi = \\ & = \exp \left(-\frac{ik}{2z} x^2 \right) \int_{-a/2}^{+a/2} \exp \left\{ -\frac{ik}{2} \left[\xi^2 \left(\frac{1}{l_E} + \frac{1}{z} \right) - \frac{2x\xi}{z} \right] \right\} d\xi \end{aligned} \quad (12.2.4)$$

Aggiungendo e sottraendo la quantità $\frac{x^2}{z^2 \left(\frac{1}{l_E} + \frac{1}{z} \right)}$ si ha:

$$\begin{aligned} & \exp \left\{ -\frac{ik}{2} \left[\xi^2 \left(\frac{1}{l_E} + \frac{1}{z} \right) - \frac{2x\xi}{z} \right] \right\} = \exp \left[-\frac{ik}{2} \left(\xi \sqrt{\frac{1}{l_E} + \frac{1}{z}} - \frac{x}{z \sqrt{\frac{1}{l_E} + \frac{1}{z}}} \right)^2 \right] \cdot \\ & \cdot \exp \left[\frac{ik}{2} \frac{x^2}{z^2 \left(\frac{1}{l_E} + \frac{1}{z} \right)} \right] \end{aligned} \quad (12.2.5)$$

Quindi l'equazione (12.2.4) diventa:

$$\exp \left\{ \frac{ik}{2z} x^2 \left[1 - \frac{1}{z \left(\frac{1}{l_E} + \frac{1}{z} \right)} \right] \right\} \int_{-a}^{+a} \exp \left[-\frac{i\pi}{\lambda} \left(\xi \sqrt{\frac{1}{l_E} + \frac{1}{z}} - \frac{x}{z \sqrt{\frac{1}{l_E} + \frac{1}{z}}} \right)^2 \right] d\xi \quad (12.2.6)$$

avendo posto dentro l'integrale $k = \frac{2\pi}{\lambda}$.

Poniamo:

$$u = \sqrt{\frac{2}{\lambda}} \left(\xi \sqrt{\frac{1}{l_E} + \frac{1}{z}} - \frac{x}{z \sqrt{\frac{1}{l_E} + \frac{1}{z}}} \right) \quad (12.2.7)$$

$$du = \sqrt{\frac{2}{\lambda}} \sqrt{\frac{1}{l_E} + \frac{1}{z}} d\xi \implies d\xi = \sqrt{\frac{\lambda}{2}} \sqrt{\frac{l_E z}{l_E + z}} du \quad (12.2.8)$$

L'equazione (12.6.6) si scrive, allora:

$$\exp \left\{ \frac{ik}{2z} x^2 \left[1 - \frac{1}{z \left(\frac{1}{l_E} + \frac{1}{z} \right)} \right] \right\} \sqrt{\frac{\lambda}{2}} \sqrt{\frac{l_E z}{l_E + z}} \int_{u_1}^{u_2} \exp \left(-\frac{i\pi}{2} u^2 \right) du \quad (12.2.9)$$

ossia:

$$\exp \left\{ \frac{ik}{2z} x^2 \left[1 - \frac{1}{z \left(\frac{1}{l_E} + \frac{1}{z} \right)} \right] \right\} \sqrt{\frac{\lambda}{2}} \sqrt{\frac{l_E z}{l_E + z}} \left[\mathcal{F}(u_2) + \mathcal{F}(-u_1) \right] \quad (12.2.10)$$

Il secondo integrale della (12.2.3) si può scrivere:

$$\begin{aligned} & \int_{-b/2}^{+b/2} \cos \left(\frac{\pi\eta}{b} \right) \exp \left[-\frac{ik}{2} \left(\frac{\eta^2}{l_H} \right) \right] \exp \left[-\frac{ik}{2z} (y^2 - 2y\eta + \eta^2) \right] d\eta = \\ & = \frac{1}{2} \int_{-b/2}^{+b/2} \left[\exp \left(\frac{i\pi\eta}{b} \right) + \exp \left(-\frac{i\pi\eta}{b} \right) \right] \exp \left[-\frac{ik}{2} \left(\frac{\eta^2}{l_H} \right) \right] \cdot \\ & \cdot \exp \left[-\frac{ik}{2z} (y^2 - 2y\eta + \eta^2) \right] d\eta = \quad (12.2.11) \\ & = \frac{1}{2} \int_{-b/2}^{+b/2} \exp \left(\frac{i\pi\eta}{b} \right) \exp \left[-\frac{ik}{2} \left(\frac{\eta^2}{l_H} \right) \right] \exp \left[-\frac{ik}{2z} (y^2 - 2y\eta + \eta^2) \right] d\eta + \\ & + \frac{1}{2} \int_{-b/2}^{+b/2} \exp \left(-\frac{i\pi\eta}{b} \right) \exp \left[-\frac{ik}{2} \left(\frac{\eta^2}{l_H} \right) \right] \exp \left[-\frac{ik}{2z} (y^2 - 2y\eta + \eta^2) \right] d\eta \end{aligned}$$

Trasformiamo il primo dei due ultimi integrali.

$$\begin{aligned}
 & \frac{1}{2} \int_{-b/2}^{+b/2} \exp\left(\frac{i\pi\eta}{b}\right) \exp\left[-\frac{ik}{2} \left(\frac{\eta^2}{l_H}\right)\right] \exp\left[-\frac{ik}{2z} (y^2 - 2y\eta + \eta^2)\right] d\eta = \\
 &= \frac{1}{2} \int_{-b/2}^{+b/2} \exp\left\{-i \left[\frac{k}{2z} y^2 - \frac{k}{z} y\eta + \frac{k}{2z} \eta^2 + \frac{k}{2l_H} \eta^2 - \frac{\pi}{b} \eta\right]\right\} d\eta = \\
 &= \frac{1}{2} \int_{-b/2}^{+b/2} \exp\left(-i \frac{k}{2z} y^2\right) \exp\left\{-i \left[-\frac{k}{z} y\eta + \frac{k}{2z} \eta^2 + \frac{k}{2l_H} \eta^2 - \frac{\pi}{b} \eta\right]\right\} d\eta = \\
 &= \frac{1}{2} \exp\left(-i \frac{k}{2z} y^2\right) \int_{-b/2}^{+b/2} \exp\left\{-i \left[\eta^2 \frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right) - 2\eta \left(\frac{ky}{2z} + \frac{\pi}{2b}\right)\right]\right\} d\eta
 \end{aligned} \tag{12.2.12}$$

Aggiungendo e sottraendo all'interno dell'integrale la quantità:

$$\frac{\left(\frac{ky}{2z} + \frac{\pi}{2b}\right)^2}{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)} \tag{12.2.13}$$

la quantità dentro le parentesi quadre all'interno dell'integrale si può scrivere:

$$\begin{aligned}
 & \left[\eta^2 \frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right) - 2\eta \left(\frac{ky}{2z} + \frac{\pi}{2b}\right)\right] + \frac{\left(\frac{ky}{2z} + \frac{\pi}{2b}\right)^2}{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)} - \frac{\left(\frac{ky}{2z} + \frac{\pi}{2b}\right)^2}{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)} = \\
 &= \left[\eta \sqrt{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)} - \frac{\left(\frac{ky}{2z} + \frac{\pi}{2b}\right)}{\sqrt{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)}}\right]^2 - \frac{\left(\frac{ky}{2z} + \frac{\pi}{2b}\right)^2}{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)}
 \end{aligned} \tag{12.2.14}$$

Quindi la (12.6.12) si può scrivere:

$$\begin{aligned}
 & \frac{1}{2} \exp\left(-i \frac{k}{2z} y^2\right) \exp\left[+i \frac{\left(\frac{ky}{2z} + \frac{\pi}{2b}\right)^2}{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)}\right] \cdot \\
 & \cdot \int_{-b/2}^{+b/2} \exp\left\{-i \left[\eta \sqrt{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)} - \frac{\left(\frac{ky}{2z} + \frac{\pi}{2b}\right)}{\sqrt{\frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right)}} \right]^2\right\} d\eta = \\
 & = \frac{1}{2} \exp\left(-i \frac{k}{2z} y^2\right) \exp\left[+i \frac{k \left(\frac{y}{z} + \frac{\pi}{kb}\right)^2}{\frac{1}{z} + \frac{1}{l_H}}\right] \cdot \\
 & \cdot \int_{-b/2}^{+b/2} \exp\left\{-i \frac{k}{2} \left(\frac{1}{z} + \frac{1}{l_H}\right) \left[\eta - \frac{\left(\frac{y}{z} + \frac{\pi}{kb}\right)}{\left(\frac{1}{z} + \frac{1}{l_H}\right)} \right]^2\right\} d\eta
 \end{aligned} \tag{12.2.15}$$

Poniamo:

$$\nu = \sqrt{\frac{2}{\lambda} \left(\frac{1}{z} + \frac{1}{l_H}\right)} \left[\eta - \frac{\left(\frac{y}{z} + \frac{\pi}{kb}\right)}{\left(\frac{1}{z} + \frac{1}{l_H}\right)} \right] \tag{12.2.16}$$

$$d\nu = \sqrt{\frac{2}{\lambda} \left(\frac{1}{z} + \frac{1}{l_H}\right)} d\eta \implies d\eta = \sqrt{\frac{\lambda}{2} \left(\frac{z l_H}{l_H + z}\right)} d\nu \tag{12.2.17}$$

Quindi la (12.2.15) diventa:

$$\frac{1}{2} \exp\left(-i \frac{k}{2z} y^2\right) \exp\left[+i \frac{k \left(\frac{y}{z} + \frac{\pi}{kb}\right)^2}{\frac{1}{z} + \frac{1}{l_H}}\right] \sqrt{\frac{\lambda}{2} \left(\frac{z l_H}{l_H + z}\right)} \int_{-\nu_1}^{+\nu_2} \exp\left(-i \frac{\pi}{2} \nu^2\right) d\nu \tag{12.2.18}$$

ossia:

$$\frac{1}{2} \exp\left(-i \frac{k}{2z} y^2\right) \exp\left[+i \frac{k \left(\frac{y}{z} + \frac{\pi}{kb}\right)^2}{\frac{1}{z} + \frac{1}{l_H}}\right] \sqrt{\frac{\lambda}{2} \left(\frac{z l_H}{l_H + z}\right)} \left[\mathcal{F}(\nu_2) + \mathcal{F}(-\nu_1) \right] \tag{12.2.19}$$

Con analoghi passaggi risolviamo il secondo integrale della (12.2.11):

$$\frac{1}{2} \exp\left(-i\frac{k}{2z}y^2\right) \exp\left[+i\frac{k}{2}\frac{\left(\frac{y}{z} - \frac{\pi}{kb}\right)}{\frac{1}{z} + \frac{1}{l_H}}\right] \sqrt{\frac{\lambda}{2}\left(\frac{zl_H}{l_H+z}\right)} \left[\mathcal{F}(w_2) - \mathcal{F}(-w_1)\right] \quad (12.2.20)$$

Diagrammi di radiazione 3D
Antenna Piramidale: Diffrazione di Fresnel ($z=0.25\lambda$)

$\nu=10$ GHz, $\lambda=3$ cm, $a=1\lambda$, $b=a$, $l=10\lambda$, $z=0.25\lambda$

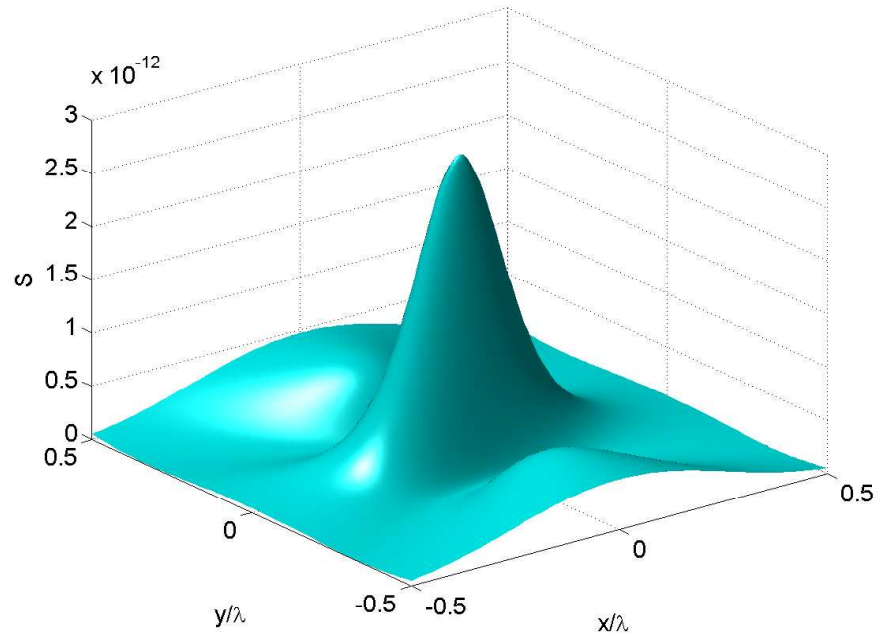


fig.12.2-1

$\nu=10$ GHz, $\lambda=3$ cm, $a=2\lambda$, $b=a$, $l=10\lambda$, $z=0.25\lambda$

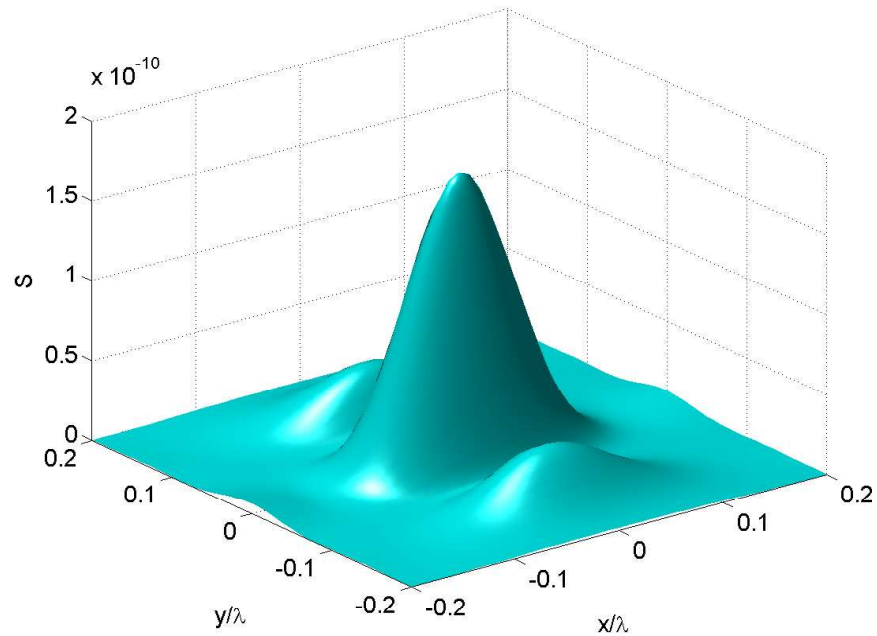


fig.12.2-2

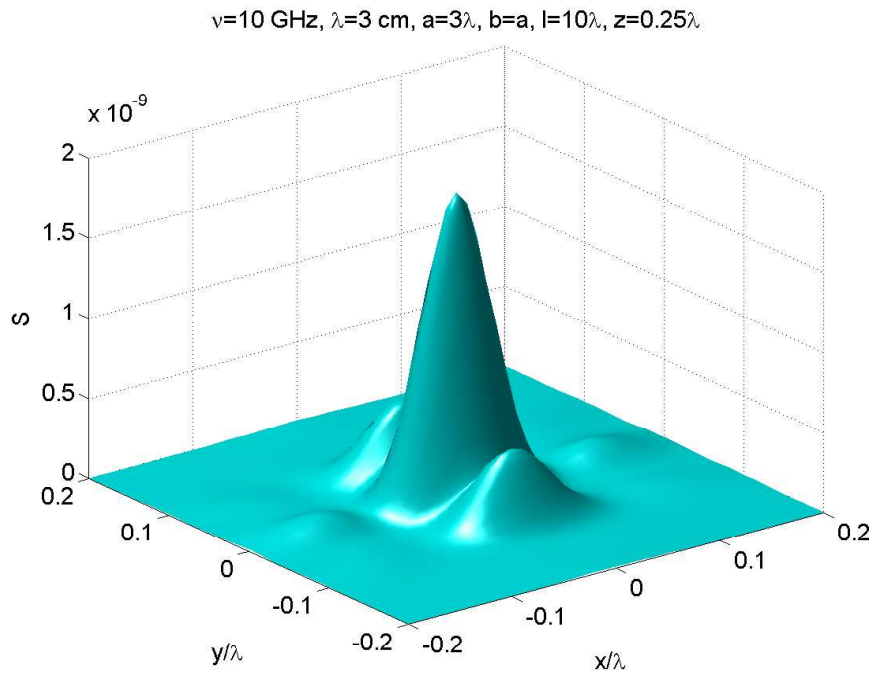


fig.12.2-3

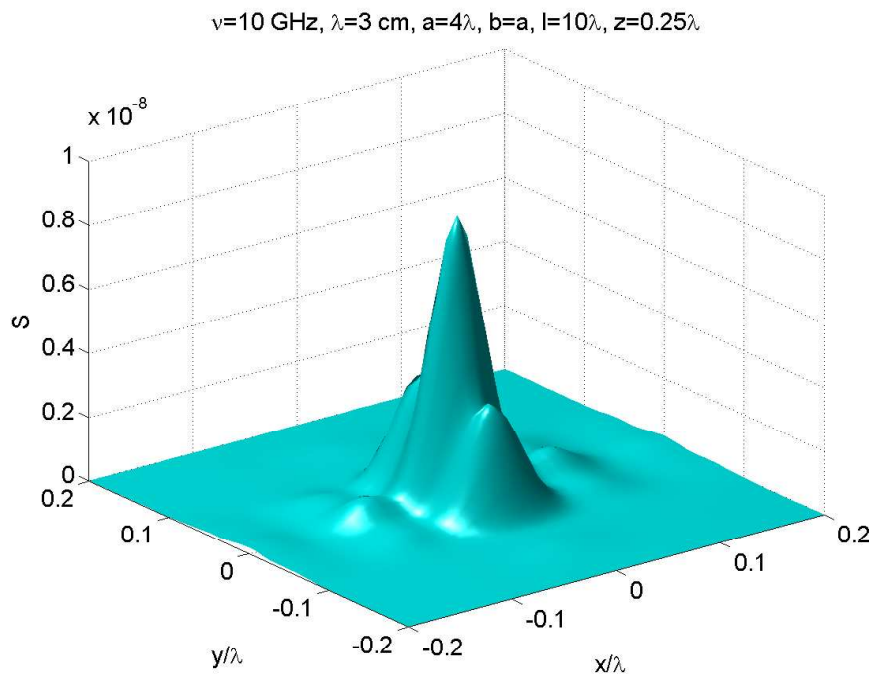


fig.12.2-4

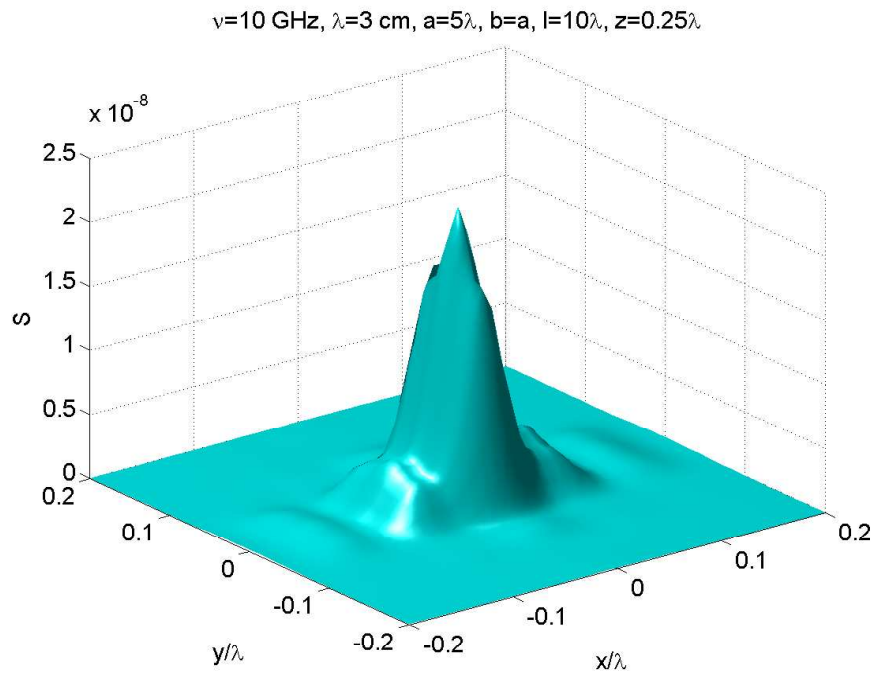


fig.12.2-5

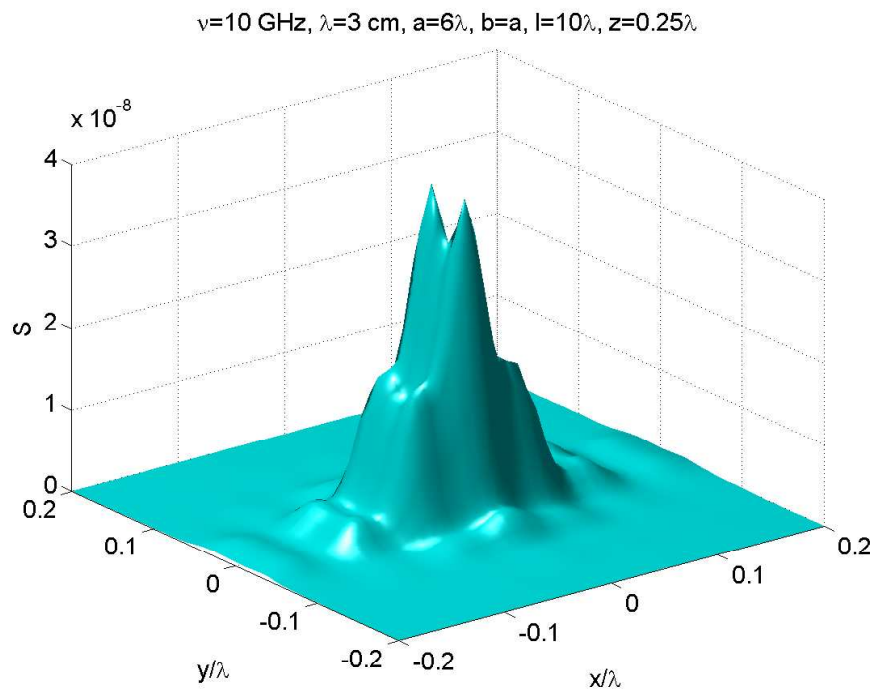


fig.12.2-6

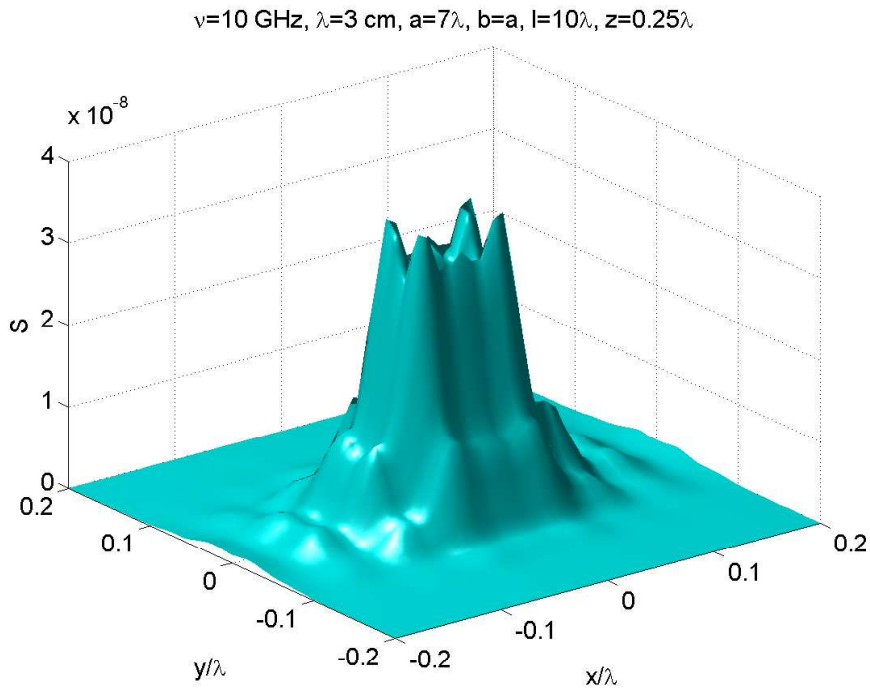


fig.12.2-7

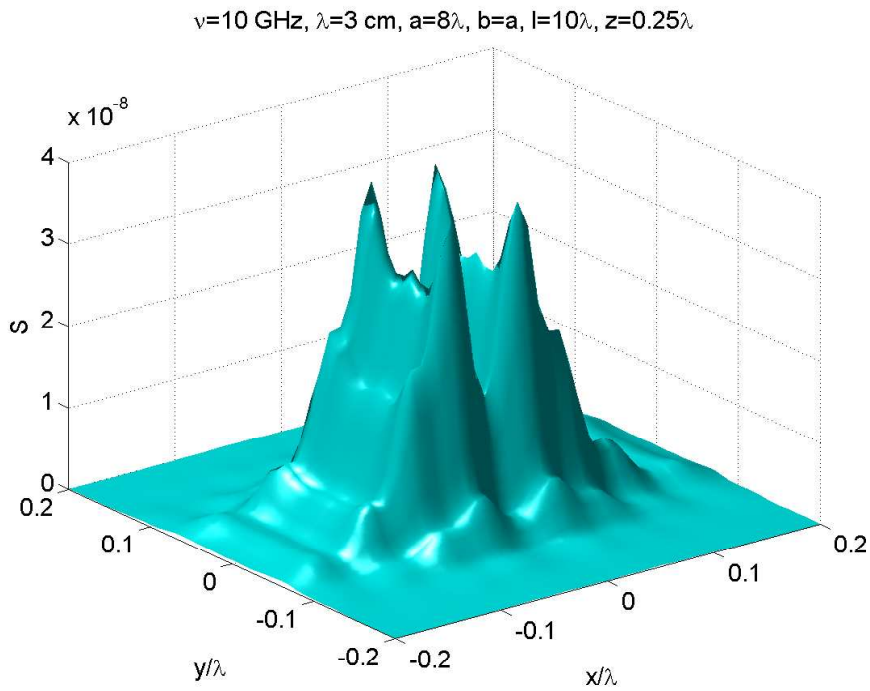


fig.12.2-8

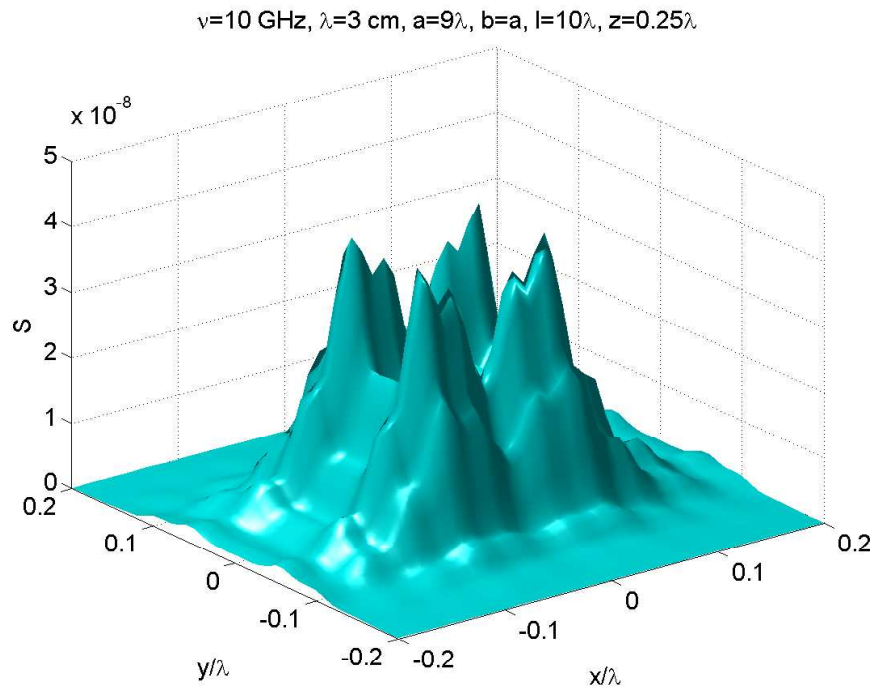


fig.12.2-9

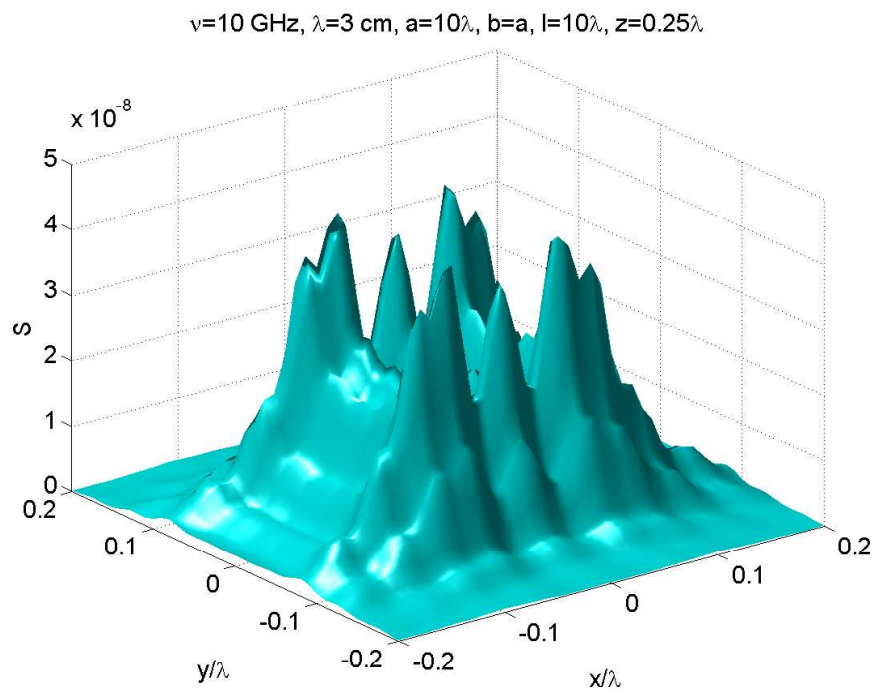


fig.12.2-10

Horn Piramidale: Diffrazione di Fresnel Programma 3D in ambiente Matlab

```
01) - %%%% Diffrazione di Fresnel da horn piramidale %%%%
02) - delete(get(0,'children'));
03) - clear all;
04) - freq=10e9;%%%% frequenza della radiazione emessa %%%%
05) - c=3e8;
06) - lambda=c./freq;%%%% lunghezza d'onda %%%%
07) - a=1.*lambda;
08) - b=a;
09) - E0=1;
10) - Z=377;
11) - k=2.*pi./lambda;
12) - lE=10.*lambda;
13) - lH=lE;
14) - [x,y] = meshgrid([-5.*lambda:1.*lambda:+5.*lambda]);
15) - z=0.25.*lambda;
16) - r=sqrt(x.^2+y.^2+z.^2);
17) - alfa=x./r;
18) - beta=y./r;
19) - gamma=z./r;
20) - u1=-(sqrt(2./lambda./lE).*a./2+(2.*alfa./lambda).*sqrt(lambda.*lE./2));
21) - u2=+(sqrt(2./lambda./lE).*a./2-(2.*alfa./lambda).*sqrt(lambda.*lE./2));
22) - v1=-(sqrt(2./lambda./lH).*b./2+(2.*beta./lambda+1./b).*
sqrt(lambda.*lH./2));
23) - v2=+(sqrt(2./lambda./lH).*b./2-(2.*beta./lambda+1./b).*
sqrt(lambda.*lH./2));
24) - w1=-(sqrt(2./lambda./lH).*b./2+(2.*beta./lambda-1./b).*
sqrt(lambda.*lH./2));
25) - w2=+(sqrt(2./lambda./lH).*b./2-(2.*beta./lambda-1./b).*
sqrt(lambda.*lH./2));
26) - Cu1=mfun('FresnelC',u1);
27) - Su1=mfun('FresnelS',u1);
28) - Cu2=mfun('FresnelC',u2);
29) - Su2=mfun('FresnelS',u2);
30) - Cv1=mfun('FresnelC',v1);
31) - Sv1=mfun('FresnelS',v1);
32) - Cv2=mfun('FresnelC',v2);
33) - Sv2=mfun('FresnelS',v2);
34) - Cw1=mfun('FresnelC',w1);
35) - Sw1=mfun('FresnelS',w1);
36) - Cw2=mfun('FresnelC',w2);
37) - Sw2=mfun('FresnelS',w2);
```

```

38) - Fu2=Cu2-i.*Su2;
39) - Fu1=Cu1-i.*Su1;
40) - Fv2=Cv2-i.*Sv2;
41) - Fv1=Cv1-i.*Sv1;
42) - Fw2=Cw2-i.*Sw2;
43) - Fw1=Cw1-i.*Sw1;
44) - %xxxxx Funzione F_x xxxxxxxx
45) - A1=E0./(2.*lambda.^2).*sqrt(lambda.*lE./2).*
exp(i.*(pi.*alfa.^2.*lE./lambda)).*(Fu2-Fu1);
46) - A2=sqrt(lambda.*lH./2).*exp(i.*(lH.*lambda./4./pi).*
(2.*pi./lambda.*beta+pi./b).^2).*(Fv2-Fv1);
47) - A3=sqrt(lambda.*lH./2).*exp(i.*(lH.*lambda./4./pi).*
(2.*pi./lambda.*beta-pi./b).^2).*(Fw2-Fw1);
48) - Fx=A1.*(A2+A3);
49) - %xxxxx Fresnel xxxxxxx
50) - fac11=1-1./(z.*(1./lE+1./z));
51) - fac10=exp(i.*k./2./z.*x.^2.*fac11);
52) - Primointegrale=fac10.*sqrt(lambda./2).*sqrt(lE.*z./(lE+z)).*(Fu2-Fu1);
53) - fac21=exp(i.*k./2.*(y./z+pi./k./b)./(1./z+1./lH));
54) - fac20=exp(-i.*k./2./z.*y.^2).*fac21;
55) - Secondointegraleprimo=0.5.*fac20.*sqrt(lambda./2).*
sqrt(z.*lH./(lH+z)).*(Fv2-Fv1);
56) - fac22=exp(i.*k./2.*(y./z-pi./k./b)./(1./z+1./lH));
57) - fac30=exp(-i.*k./2./z.*y.^2).*fac22;
58) - Secondointegralesecondo=0.5.*fac30.*sqrt(lambda./2).*
sqrt(z.*lH./(lH+z)).*(Fw2-Fw1);
59) - S=1./2./Z.*(1./lambda.^2./z.^2).*
abs(Primointegrale.*Secondointegraleprimo.*Secondointegralesecondo).^2;
60) - surf(x./lambda, y./lambda, S, 'FaceColor', 'cyan', 'EdgeColor', 'None')
61) - camlight left; lighting phong
62) - set(gca,'fontsize',15);
63) - xlabel('x/λ');
64) - ylabel('y/λ');
65) - zlabel('S');
66) - title('ν=10 GHz, λ=3 cm, a=1λ, b=a, lE=lH=10λ, z = 0.25λ')
67) - %view([60 90])
68) - print('-depsc2','hornpiramidalefresnel_1')

```

12.3 - Antenna ad horn circolare

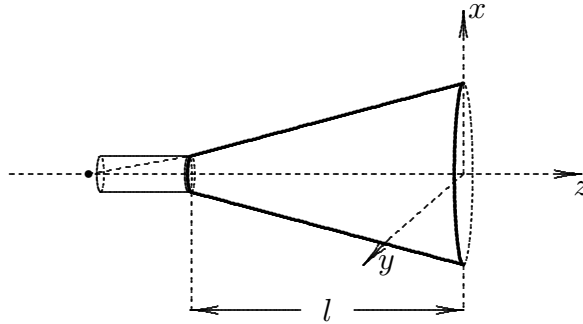


fig.12.3-1

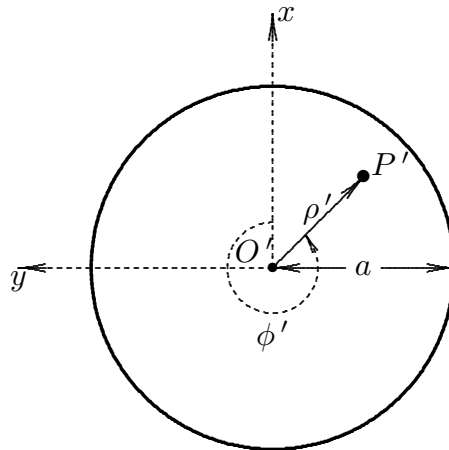


fig.12.3-2

Consideriamo un'antenna ad horn circolare alimentata da una guida d'onda eccitata nel modo TE_{11} . Procedendo come nel caso di antenna piramidale, il campo sulla bocca (apertura) dell'antenna \vec{E}_a é:

$$E_{a_{\rho'}} = \frac{i\omega\mu}{h^2} \frac{1}{\rho'} A_1 J_1 \left(\frac{x'_{11}\rho'}{a} \right) \sin(\phi' - \phi'_0) \exp\left(-\frac{ik\rho'^2}{2l}\right) \quad (12.3.1)$$

$$E_{a_{\phi'}} = \frac{i\omega\mu}{h} A_1 J'_1 \left(\frac{x'_{11}\rho'}{a} \right) \cos(\phi' - \phi'_0) \exp\left(-\frac{ik\rho'^2}{2l}\right) \quad (12.3.2)$$

Il campo elettrico diffratto dall'horn in un generico punto P dello spazio é dato in coordinate cartesiane da:

$$E_x(x, y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F_x(\alpha', \beta') \exp\{-ikr(\alpha'\alpha + \beta'\beta + \gamma'\gamma)\} d\alpha' d\beta' \quad (12.3.3)$$

$$E_y(x, y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F_y(\alpha', \beta') \exp\{-ikr(\alpha'\alpha + \beta'\beta + \gamma'\gamma)\} d\alpha' d\beta' \quad (12.3.4)$$

Le funzioni spettro angolare F_x e F_y sono:

$$F_x(\alpha', \beta') = \frac{1}{\lambda^2} \iint_{\mathbb{R}^2} E_{a_x'}(x', y') \exp[ik(\alpha'x + \beta'y)] dx dy \quad (12.3.5)$$

$$F_y(\alpha', \beta') = \frac{1}{\lambda^2} \iint_{\mathbb{R}^2} E_{a_y'}(x', y') \exp[ik(\alpha'x + \beta'y)] dx dy \quad (12.3.6)$$

Utilizziamo le seguenti trasformazioni:

$$E_{a_x'} = \cos \phi' E_{\rho'} - \sin \phi' E_{\phi'} \quad (12.3.7)$$

$$E_{a_y'} = \sin \phi' E_{\rho'} + \cos \phi' E_{\phi'} \quad (12.3.8)$$

Quindi:

$$\begin{aligned} E_{a_x'} = & \cos \phi' \left[\frac{i\omega\mu}{h^2} \frac{1}{\rho'} A_1 J_1 \left(\frac{x'_{11}\rho'}{a} \right) \sin(\phi' - \phi'_0) \exp\left(-\frac{ik\rho'^2}{2l}\right) \right] - \\ & - \sin \phi' \left[\frac{i\omega\mu}{h} A_1 J'_1 \left(\frac{x'_{11}\rho'}{a} \right) \cos(\phi' - \phi'_0) \exp\left(-\frac{ik\rho'^2}{2l}\right) \right] \end{aligned} \quad (12.3.9)$$

Calcoliamo $F_x(\alpha', \beta')$ del primo addendo:

$$\begin{aligned} & \frac{1}{\lambda^2} \iint \left\{ \cos \phi' \frac{i\omega\mu}{h^2} \frac{1}{\rho'} A_1 J_1 \left(\frac{x'_{11}\rho'}{a} \right) \sin(\phi' - \phi'_0) \exp\left(-\frac{ik\rho'^2}{2l}\right) \right\} \cdot \\ & \cdot \exp[ik\rho' \sin \theta \cos(\phi - \phi')] \rho' d\rho' d\phi' = \\ & = \frac{i\omega\mu}{h^2 \lambda^2} A_1 \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) \exp\left(-\frac{ik\rho'^2}{2l}\right) \cdot \\ & \cdot \left\{ \int_0^{2\pi} \cos \phi' \sin(\phi' - \phi'_0) \exp[ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \right\} d\rho' \end{aligned} \quad (12.3.10)$$

Calcoliamo l'integrale dentro le parentesi graffe:

$$\int_0^{2\pi} \cos \phi' \sin(\phi' - \phi'_0) \exp[ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \quad (12.3.11)$$

Utilizziamo lo sviluppo di Jacobi:

$$\begin{aligned} \exp[ik\rho' \sin \theta \cos(\phi - \phi')] = & J_0(k\rho' \sin \theta) + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \cos(n\phi) \cos(n\phi') + \\ & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \sin(n\phi) \sin(n\phi') \end{aligned} \quad (12.3.12)$$

Ne segue che l'integrale (12.3.11) si scrive:

$$\begin{aligned}
 & J_0(k\rho' \sin \theta) \int_0^{2\pi} \cos \phi' \sin(\phi' - \phi'_0) d\phi' + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \cos(n\phi) \int_0^{2\pi} \cos(\phi') \sin(\phi' - \phi'_0) \cos n\phi' d\phi' + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \sin(n\phi) \int_0^{2\pi} \cos(\phi') \sin(\phi' - \phi'_0) \sin n\phi' d\phi'
 \end{aligned} \tag{12.3.13}$$

Risolviamo gli integrali in questo modo:

$$\begin{aligned}
 & \cos(\phi') \sin(\phi' - \phi'_0) \cos n\phi' = -\frac{i}{8} \left(e^{i\phi'} + e^{-i\phi'} \right) \left[e^{i(\phi' - \phi'_0)} - e^{-i(\phi' - \phi'_0)} \right] \cdot \\
 & \cdot \left(e^{in\phi'} + e^{-in\phi'} \right) = -\frac{i}{8} \left[e^{i(2\phi' - \phi'_0)} - e^{i\phi'_0} + e^{-i\phi'_0} - e^{-i(2\phi' - \phi'_0)} \right] \left(e^{in\phi'} + e^{-in\phi'} \right) = \\
 & = -\frac{i}{8} \left\{ e^{i[\phi'(n+2) - \phi'_0]} + e^{i[\phi'(2-n) - \phi'_0]} - e^{i(n\phi' + \phi'_0)} - e^{-i(n\phi' - \phi'_0)} + e^{i(n\phi' - \phi'_0)} + \right. \\
 & \left. + e^{-i(n\phi' + \phi'_0)} - e^{i[\phi'(n-2) + \phi'_0]} - e^{-i[\phi'(n+2) - \phi'_0]} \right\}
 \end{aligned} \tag{12.3.14}$$

È evidente che eseguendo l'integrale fra 0 e 2π di ciascuna espressione all'interno delle parentesi graffe si ottiene risultato nullo tranne per $n=2$; si ha, cioè:

$$\begin{aligned}
 & \int_0^{2\pi} \cos \phi' \sin(\phi' - \phi'_0) \cos n\phi' d\phi' = -\frac{i}{8} \left\{ \int_0^{2\pi} e^{-i\phi'_0} d\phi' - \int_0^{2\pi} e^{i\phi'_0} d\phi' \right\} = \\
 & = -\frac{i}{8} \left\{ 2\pi e^{-i\phi'_0} - 2\pi e^{i\phi'_0} \right\} = -\frac{i}{8} (-4i\pi \sin \phi'_0) = -\frac{1}{2} \pi \sin \phi'_0
 \end{aligned} \tag{12.3.15}$$

Analogamente si ha:

$$\begin{aligned}
 & \cos(\phi') \sin(\phi' - \phi'_0) \sin n\phi' = -\frac{1}{8} \left(e^{i\phi'} + e^{-i\phi'} \right) \left[e^{i(\phi' - \phi'_0)} - e^{-i(\phi' - \phi'_0)} \right] \cdot \\
 & \cdot \left(e^{in\phi'} - e^{-in\phi'} \right) = -\frac{1}{8} \left[e^{i(2\phi' - \phi'_0)} - e^{i\phi'_0} + e^{-i\phi'_0} - e^{-i(2\phi' - \phi'_0)} \right] \left(e^{in\phi'} - e^{-in\phi'} \right) = \\
 & = -\frac{1}{8} \left\{ e^{i[\phi'(n+2) - \phi'_0]} - e^{i[\phi'(2-n) - \phi'_0]} - e^{i(n\phi' + \phi'_0)} + e^{-i(n\phi' - \phi'_0)} + e^{i(n\phi' - \phi'_0)} - \right. \\
 & \left. - e^{-i(n\phi' + \phi'_0)} - e^{i[\phi'(n-2) + \phi'_0]} + e^{-i[\phi'(n+2) - \phi'_0]} \right\}
 \end{aligned} \tag{12.3.16}$$

Anche in questo caso é evidente che eseguendo l'integrale fra 0 e 2π di ciascuna espressione all'interno delle parentesi graffe si ottiene risultato nullo tranne per $n=2$; si ha, cioè:

$$\begin{aligned} \int_0^{2\pi} \cos \phi' \sin(\phi' - \phi'_0) \sin n\phi' d\phi' &= \frac{1}{8} \left\{ \int_0^{2\pi} e^{-i\phi'_0} d\phi' + \int_0^{2\pi} e^{i\phi'_0} d\phi' \right\} = \\ &= \frac{1}{8} \left\{ 2\pi e^{-i\phi'_0} + 2\pi e^{i\phi'_0} \right\} = \frac{1}{8} (4\pi \cos \phi'_0) = \frac{1}{2} \pi \cos \phi'_0 \end{aligned} \quad (12.3.17)$$

In definitiva l'integrale (12.3.11) si scrive:

$$\begin{aligned} & - \pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \pi \sin \phi'_0 \cos 2\phi J_2(k\rho' \sin \theta) - \pi \cos \phi'_0 \sin 2\phi J_2(k\rho' \sin \theta) = \\ & = -\pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \pi J_2(k\rho' \sin \theta) [\sin \phi'_0 \cos 2\phi - \cos \phi'_0 \sin 2\phi] = \\ & = -\pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \pi J_2(k\rho' \sin \theta) \sin(\phi'_0 - 2\phi) \end{aligned} \quad (12.3.18)$$

La prima parte di F_x é quindi:

$$\begin{aligned} & \frac{i\omega\mu}{h^2\lambda^2} A_1 \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) \exp \left(-\frac{ik\rho'^2}{2l} \right) \left\{ -\pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \right. \\ & \left. + \pi \sin(\phi'_0 - 2\phi) J_2(k\rho' \sin \theta) \right\} d\rho' = \\ & = -\frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \sin \phi'_0 \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' + \\ & + \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \sin(\phi'_0 - 2\phi) \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' \end{aligned} \quad (12.3.19)$$

La seconda parte di F_x é:

$$\begin{aligned} & -\frac{1}{\lambda^2} \iint \sin \phi' \frac{i\omega\mu}{h} A_1 J'_1 \left(\frac{x'_{11}\rho'}{a} \right) \cos(\phi' - \phi'_0) \exp \left(-\frac{ik\rho'^2}{2l} \right) \cdot \\ & \cdot \exp[ik\rho' \sin \theta \cos(\phi - \phi')] \rho' d\rho' d\phi' = \\ & = -\frac{i\omega\mu}{h\lambda^2} A_1 \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) \exp \left(-\frac{ik\rho'^2}{2l} \right) \cdot \\ & \cdot \left\{ \int_0^{2\pi} \sin \phi' \cos(\phi' - \phi'_0) \exp[ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \right\} \rho' d\rho' \end{aligned} \quad (12.3.20)$$

Calcoliamo l'integrale dentro le parentesi graffe:

$$\int_0^{2\pi} \sin \phi' \cos(\phi' - \phi'_0) \exp[ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \quad (12.3.21)$$

Utilizzando la (12.3.12), la (12.3.21) si scrive:

$$\begin{aligned}
 & J_0(k\rho' \sin \theta) \int_0^{2\pi} \sin \phi' \cos(\phi' - \phi'_0) d\phi' + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \cos(n\phi) \int_0^{2\pi} \sin(\phi') \cos(\phi' - \phi'_0) \cos n\phi' d\phi' + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \sin(n\phi) \int_0^{2\pi} \sin(\phi') \cos(\phi' - \phi'_0) \sin n\phi' d\phi' = \\
 & = \pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \pi \sin \phi'_0 \cos 2\phi J_2(k\rho' \sin \theta) - \pi \cos \phi'_0 \sin 2\phi J_2(k\rho' \sin \theta) = \\
 & = \pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \pi J_2(k\rho' \sin \theta) [\sin \phi'_0 \cos 2\phi - \cos \phi'_0 \sin 2\phi] = \\
 & = \pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \pi J_2(k\rho' \sin \theta) \sin(\phi'_0 - 2\phi)
 \end{aligned} \tag{12.3.22}$$

La seconda parte di F_x é quindi:

$$\begin{aligned}
 & - \frac{i\omega\mu}{h\lambda^2} A_1 \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) \exp \left(-\frac{ik\rho'^2}{2l} \right) \left\{ \pi \sin \phi'_0 J_0(k\rho' \sin \theta) + \right. \\
 & \left. + \pi \sin(\phi'_0 - 2\phi) J_2(k\rho' \sin \theta) \right\} \rho' d\rho' = \\
 & = - \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin \phi'_0 \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' - \\
 & - \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin(\phi'_0 - 2\phi) \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho'
 \end{aligned} \tag{12.3.23}$$

L'espressione finale di $F_x(\alpha, \beta)$ é, quindi:

$$\begin{aligned}
 F_x = & -\frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \sin \phi'_0 \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' + \\
 & + \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \sin(\phi'_0 - 2\phi) \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' - \\
 & - \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin \phi'_0 \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' - \\
 & - \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin(\phi'_0 - 2\phi) \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho'
 \end{aligned} \tag{12.3.24}$$

Calcoliamo la funzione spettrale $F_y(\alpha, \beta)$. Essa é:

$$F_y(\alpha, \beta) = \frac{1}{\lambda^2} \iint_{\mathbb{R}^2} E_{ax}(x, y) \exp [ik(\alpha x + \beta y)] dx dy \tag{12.3.25}$$

Poiché:

$$E_y = \sin \phi' E_{\rho'} + \cos \phi' E_{\phi'} \tag{12.3.26}$$

$$\begin{aligned}
 E_y = \sin \phi' & \left[\frac{i\omega\mu}{h^2} \frac{1}{\rho'} A_1 J_1 \left(\frac{x'_{11}\rho'}{a} \right) \sin(\phi' - \phi'_0) \exp \left(-\frac{ik\rho'^2}{2l} \right) \right] + \\
 & + \cos \phi' \left[\frac{i\omega\mu}{h} A_1 J'_1 \left(\frac{x'_{11}\rho'}{a} \right) \cos(\phi' - \phi'_0) \exp \left(-\frac{ik\rho'^2}{2l} \right) \right]
 \end{aligned} \tag{12.3.27}$$

Calcoliamo $F_y(\alpha, \beta)$ del primo addendo:

$$\begin{aligned}
 & \frac{1}{\lambda^2} \iint \left\{ \sin \phi' \frac{i\omega\mu}{h^2} \frac{1}{\rho'} A_1 J_1 \left(\frac{x'_{11}\rho'}{a} \right) \sin(\phi' - \phi'_0) \exp \left(-\frac{ik\rho'^2}{2l} \right) \right\} \cdot \\
 & \cdot \exp [ik\rho' \sin \theta \cos(\phi - \phi')] \rho' d\rho' d\phi' = \\
 & = \frac{i\omega\mu}{h^2\lambda^2} A_1 \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) \exp \left(-\frac{ik\rho'^2}{2l} \right) \cdot \\
 & \cdot \left\{ \int_0^{2\pi} \sin \phi' \sin(\phi' - \phi'_0) \exp [ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \right\} d\rho'
 \end{aligned} \tag{12.3.28}$$

Calcoliamo l'integrale dentro le parentesi graffe:

$$\int_0^{2\pi} \sin \phi' \sin(\phi' - \phi'_0) \exp[ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \quad (12.3.29)$$

Utilizzando lo sviluppo di Jacobi (12.3.9), l'integrale (12.3.24) si scrive:

$$\begin{aligned} & J_0(k\rho' \sin \theta) \int_0^{2\pi} \sin \phi' \sin(\phi' - \phi'_0) d\phi' + \\ & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \cos(n\phi) \int_0^{2\pi} \sin(\phi') \sin(\phi' - \phi'_0) \cos n\phi' d\phi' + \\ & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \sin(n\phi) \int_0^{2\pi} \sin(\phi') \sin(\phi' - \phi'_0) \sin n\phi' d\phi' = \\ & = \pi \cos \phi'_0 J_0(k\rho' \sin \theta) + \pi \cos \phi'_0 \cos 2\phi J_2(k\rho' \sin \theta) + \pi \sin \phi'_0 \sin 2\phi J_2(k\rho' \sin \theta) = \\ & = \pi \cos \phi'_0 J_0(k\rho' \sin \theta) + \pi J_2(k\rho' \sin \theta) [\sin \phi'_0 \cos 2\phi + \cos \phi'_0 \cos 2\phi] = \\ & = \pi \cos \phi'_0 J_0(k\rho' \sin \theta) + \pi J_2(k\rho' \sin \theta) \cos(\phi'_0 - 2\phi) \end{aligned} \quad (12.3.30)$$

La prima parte di F_y é quindi:

$$\begin{aligned} & \frac{i\omega\mu}{h^2\lambda^2} A_1 \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) \exp\left(-\frac{ik\rho'^2}{2l}\right) \left\{ \pi \cos \phi'_0 J_0(k\rho' \sin \theta) + \right. \\ & \left. + \pi \cos(\phi'_0 - 2\phi) J_2(k\rho' \sin \theta) \right\} d\rho' = \\ & = \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \cos \phi'_0 \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\ & + \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \cos(\phi'_0 - 2\phi) \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' \end{aligned} \quad (12.3.31)$$

La seconda parte di F_y é:

$$\begin{aligned} & \frac{1}{\lambda^2} \iint \cos \phi' \frac{i\omega\mu}{h} A_1 J'_1\left(\frac{x'_{11}\rho'}{a}\right) \cos(\phi' - \phi'_0) \exp\left(-\frac{ik\rho'^2}{2l}\right) \cdot \\ & \cdot \exp[ik\rho' \sin \theta \cos(\phi - \phi')] \rho' d\rho' d\phi' = \\ & = + \frac{i\omega\mu}{h\lambda^2} A_1 \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) \exp\left(-\frac{ik\rho'^2}{2l}\right) \cdot \\ & \cdot \left\{ \int_0^{2\pi} \cos \phi' \cos(\phi' - \phi'_0) \exp[ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \right\} \rho' d\rho' \end{aligned} \quad (12.3.32)$$

Calcoliamo l'integrale dentro le parentesi graffe:

$$\int_0^{2\pi} \cos \phi' \cos(\phi' - \phi'_0) \exp[ik\rho' \sin \theta \cos(\phi - \phi')] d\phi' \quad (12.3.33)$$

Utilizzando la (12.3.9), la (12.3.31) si scrive:

$$\begin{aligned} & J_0(k\rho' \sin \theta) \int_0^{2\pi} \cos \phi' \cos(\phi' - \phi'_0) d\phi' + \\ & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \cos(n\phi) \int_0^{2\pi} \cos \phi' \cos(\phi' - \phi'_0) \cos n\phi' d\phi' + \\ & + 2 \sum_{n=1}^{+\infty} i^n J_n(k\rho' \sin \theta) \sin(n\phi) \int_0^{2\pi} \cos \phi' \cos(\phi' - \phi'_0) \sin n\phi' d\phi' = \\ & = \pi \cos \phi'_0 J_0(k\rho' \sin \theta) - \pi \cos \phi'_0 \cos 2\phi J_2(k\rho' \sin \theta) - \pi \sin \phi'_0 \sin 2\phi J_2(k\rho' \sin \theta) = \\ & = \pi \cos \phi'_0 J_0(k\rho' \sin \theta) - \pi J_2(k\rho' \sin \theta) [\cos \phi'_0 \cos 2\phi + \sin \phi'_0 \sin 2\phi] = \\ & = \pi \cos \phi'_0 J_0(k\rho' \sin \theta) - \pi J_2(k\rho' \sin \theta) \cos(\phi'_0 - 2\phi) \end{aligned} \quad (12.3.34)$$

La seconda parte di F_y é quindi:

$$\begin{aligned} & \frac{i\omega\mu}{h\lambda^2} A_1 \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) \exp \left(-\frac{ik\rho'^2}{2l} \right) \left\{ \pi \cos \phi'_0 J_0(k\rho' \sin \theta) - \right. \\ & \left. - \pi \cos(\phi'_0 - 2\phi) J_2(k\rho' \sin \theta) \right\} \rho' d\rho' = \\ & = \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \phi'_0 \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' - \\ & - \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos(\phi'_0 - 2\phi) \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' \end{aligned} \quad (12.3.35)$$

L'espressione finale di $F_y(\alpha, \beta)$ é, quindi:

$$\begin{aligned}
 F_y = & \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \cos(\phi'_0 - 2\phi) \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' - \\
 & - \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos(\phi'_0 - 2\phi) \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \phi'_0 \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \phi'_0 \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho'
 \end{aligned}
 \tag{12.3.36}$$

12.4 - Valutazione della componente del campo elettrico lungo la direzione

\hat{e}_θ

Calcoliamo la componente lungo \hat{e}_θ del campo elettrico. Essa é:

$$E_\theta = \cos \phi F_x(\alpha, \beta) + \sin \phi F_y(\alpha, \beta) \tag{12.4.1}$$

ossia:

$$\begin{aligned}
 & -\frac{i\omega\mu}{h^2\lambda^2}A_1\pi\cos\phi\sin\phi'_0\int_0^aJ_1\left(\frac{x'_{11}\rho'}{a}\right)J_0(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)d\rho'+ \\
 & +\frac{i\omega\mu}{h^2\lambda^2}A_1\pi\cos\phi\sin(\phi'_0-2\phi)\int_0^aJ_1\left(\frac{x'_{11}\rho'}{a}\right)J_2(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)d\rho'- \\
 & -\frac{i\omega\mu}{h\lambda^2}A_1\pi\cos\phi\sin\phi'_0\int_0^aJ'_1\left(\frac{x'_{11}\rho'}{a}\right)J_0(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)\rho'd\rho'- \\
 & -\frac{i\omega\mu}{h\lambda^2}A_1\pi\cos\phi\sin(\phi'_0-2\phi)\int_0^aJ'_1\left(\frac{x'_{11}\rho'}{a}\right)J_2(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)\rho'd\rho'+ \\
 & +\frac{i\omega\mu}{h^2\lambda^2}A_1\pi\sin\phi\cos\phi'_0\int_0^aJ_1\left(\frac{x'_{11}\rho'}{a}\right)J_0(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)d\rho'+ \\
 & +\frac{i\omega\mu}{h^2\lambda^2}A_1\pi\sin\phi\cos(\phi'_0-2\phi)\int_0^aJ_1\left(\frac{x'_{11}\rho'}{a}\right)J_2(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)d\rho'+ \\
 & +\frac{i\omega\mu}{h\lambda^2}A_1\pi\sin\phi\cos\phi'_0\int_0^aJ'_1\left(\frac{x'_{11}\rho'}{a}\right)J_0(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)\rho'd\rho'- \\
 & -\frac{i\omega\mu}{h\lambda^2}A_1\pi\sin\phi\cos(\phi'_0-2\phi)\int_0^aJ'_1\left(\frac{x'_{11}\rho'}{a}\right)J_2(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)\rho'd\rho'
 \end{aligned} \tag{12.4.2}$$

Mettendo in evidenza gli integrale comuni, l'espressione (12.4.2) conterrà quattro termini.

$$\begin{aligned}
 & \frac{i\omega\mu}{h^2\lambda^2}A_1\pi[\sin\phi\cos\phi'_0-\cos\phi\sin\phi'_0]\int_0^aJ_1\left(\frac{x'_{11}\rho'}{a}\right)J_0(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)d\rho'+ \\
 & +\frac{i\omega\mu}{h\lambda^2}A_1\pi[\sin\phi\cos\phi'_0-\cos\phi\sin\phi'_0]\int_0^aJ'_1\left(\frac{x'_{11}\rho'}{a}\right)J_0(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)\rho'd\rho'+ \\
 & +\frac{i\omega\mu}{h^2\lambda^2}A_1\pi[\cos\phi\sin(\phi'_0-2\phi)+\sin\phi\cos(\phi'_0-2\phi)]\cdot \\
 & \cdot\int_0^aJ_1\left(\frac{x'_{11}\rho'}{a}\right)J_2(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)d\rho'+ \\
 & -\frac{i\omega\mu}{h\lambda^2}A_1\pi[\cos\phi\sin(\phi'_0-2\phi)+\sin\phi\cos(\phi'_0-2\phi)]\cdot \\
 & \cdot\int_0^aJ'_1\left(\frac{x'_{11}\rho'}{a}\right)J_2(k\rho'\sin\theta)\exp\left(-\frac{ik\rho'^2}{2l}\right)\rho'd\rho'
 \end{aligned} \tag{12.4.3}$$

ossia:

$$\begin{aligned}
 & \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \sin(\phi - \phi'_0) \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin(\phi - \phi'_0) \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' + \\
 & + \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi [\cos\phi \sin(\phi'_0 - 2\phi) + \sin\phi \cos(\phi'_0 - 2\phi)] \cdot \\
 & \cdot \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' - \\
 & - \frac{i\omega\mu}{h\lambda^2} A_1 \pi [\cos\phi \sin(\phi'_0 - 2\phi) + \sin\phi \cos(\phi'_0 - 2\phi)] \cdot \\
 & \cdot \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho'
 \end{aligned} \tag{12.4.4}$$

e, ancora:

$$\begin{aligned}
 & \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin(\phi - \phi'_0) \left[\frac{1}{h} \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \right. \\
 & \left. + \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' \right] + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin(\phi'_0 - \phi) \left[\frac{1}{h} \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' - \right. \\
 & \left. - \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' \right]
 \end{aligned} \tag{12.4.5}$$

Ordinando:

$$\begin{aligned}
 & \frac{i\omega\mu}{h^2\lambda^2} A_1 \pi \sin(\phi - \phi'_0) \left[\int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' - \right. \\
 & \left. - \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' \right] + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin(\phi - \phi'_0) \left[\int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \right. \\
 & \left. + \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' \right]
 \end{aligned} \tag{12.4.6}$$

che si può ancora scrivere:

$$\begin{aligned}
 & \frac{i\omega\mu}{h\lambda^2} A_1 \pi \sin(\phi - \phi'_0) \left[\frac{1}{h} \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' + \right. \\
 & \left. + \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' \right] + \\
 & \left[-\frac{1}{h} \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' + \right. \\
 & \left. + \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' \right]
 \end{aligned} \tag{12.4.7}$$

Scriviamo la relazione di ricorrenza delle funzioni cilindriche:

$$zJ'_\nu(z) + \nu J_\nu(z) = zJ_{\nu-1}(z) \tag{12.4.8}$$

Per $\nu = 1$, si ha:

$$zJ'_1(z) + J_1(z) = zJ_0(z) \tag{12.4.9}$$

Ponendo $z = \frac{x'_{11}\rho'}{a}$ e moltiplicando per $J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \frac{a}{x'_{11}}$, la (12.4.9)

diventa:

$$\begin{aligned}
 & \rho' J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) + \\
 & + \frac{a}{x'_{11}} J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) = \\
 & = \rho' J_0 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right)
 \end{aligned} \tag{12.4.10}$$

Integrando fra 0 e a :

$$\begin{aligned}
 & \int_0^a \rho' J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' + \\
 & + \frac{1}{h} \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' = \\
 & = \int_0^a \rho' J_0 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho'
 \end{aligned} \tag{12.4.11}$$

I primi due integrali della (12.4.11) coincidono con il termine fra parentesi quadre della (12.4.7).

Calcoliamo, quindi:

$$\int_0^a \rho' J_0\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' \quad (12.4.12)$$

Poniamo $\frac{\rho'}{a} = w$ da cui $\rho' = wa$ ossia $d\rho' = adw$. Se $\rho' = 0 \implies w = 0$; se $\rho' = a \implies w = 1$. Quindi l'integrale (12.4.12) si può scrivere:

$$\begin{aligned} & \int_0^1 wa J_0(x'_{11}w) J_0(kaw \sin \theta) \exp\left(-\frac{ika^2w^2}{2l}\right) adw = \\ & = a^2 \int_0^1 w J_0(x'_{11}w) J_0(kaw \sin \theta) \exp\left(-\frac{ika^2w^2}{2l}\right) dw \end{aligned} \quad (12.4.13)$$

Cerchiamo adesso di valutare l'espressione all'interno delle seconde parentesi quadre della (12.4.7). Consideriamo l'espressione di ricorrenza delle funzioni cilindriche:

$$zJ'_\nu(z) - \nu J_\nu(z) = -zJ_{\nu+1}(z) \quad (12.4.14)$$

Per $\nu = 1$, si ha:

$$zJ'_1(z) - J_1(z) = -zJ_2(z) \quad (12.4.15)$$

Ponendo $z = \frac{x'_{11}\rho'}{a}$ e moltiplicando per $J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \frac{a}{x'_{11}}$, la (12.4.15)

diventa:

$$\begin{aligned} & \rho' J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) - \\ & - \frac{a}{x'_{11}} J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) = \\ & = -\rho' J_2\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \end{aligned} \quad (12.4.16)$$

Integrando fra 0 e a:

$$\begin{aligned} & \int_0^a \rho' J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' - \\ & - \frac{1}{h} \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' = \\ & = - \int_0^a \rho' J_2\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' \end{aligned} \quad (12.4.17)$$

I secondi due integrali della (12.4.17) coincidono con il secondo termine fra parentesi quadre della (12.4.7).

Quindi la componente del campo elettrico lungo la direzione \hat{e}_θ é:

$$\begin{aligned} & \frac{i\omega\mu}{\lambda^2} \frac{A_1}{h} \pi \sin(\phi - \phi'_0) a^2 \left\{ \int_0^1 w J_0(x'_{11} w) J_0(kaw \sin \theta) \exp\left(-\frac{ika^2 w^2}{2l}\right) dw - \right. \\ & \left. - \int_0^1 w J_2(x'_{11} w) J_2(kaw \sin \theta) \exp\left(-\frac{ika^2 w^2}{2l}\right) dw \right\} \end{aligned} \quad (12.4.18)$$

\hat{e}_ϕ 12.5 - Valutazione della componente del campo elettrico lungo la direzione

Calcoliamo la componente lungo \hat{e}_ϕ del campo elettrico. Essa é:

$$E_\phi = -\cos \theta \sin \phi F_x(\alpha, \beta) + \cos \theta \cos \phi F_y(\alpha, \beta) \quad (12.5.1)$$

ossia:

$$\begin{aligned} & + \frac{i\omega\mu}{h^2 \lambda^2} A_1 \pi \cos \theta \sin \phi \sin \phi'_0 \int_0^a J_1\left(\frac{x'_{11} \rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' - \\ & - \frac{i\omega\mu}{h^2 \lambda^2} A_1 \pi \cos \theta \sin \phi \sin(\phi'_0 - 2\phi) \int_0^a J_1\left(\frac{x'_{11} \rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\ & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \theta \sin \phi \sin \phi'_0 \int_0^a J'_1\left(\frac{x'_{11} \rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' + \\ & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \theta \sin \phi \sin(\phi'_0 - 2\phi) \int_0^a J'_1\left(\frac{x'_{11} \rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' + \\ & + \frac{i\omega\mu}{h^2 \lambda^2} A_1 \pi \cos \theta \cos \phi \cos \phi'_0 \int_0^a J_1\left(\frac{x'_{11} \rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\ & + \frac{i\omega\mu}{h^2 \lambda^2} A_1 \pi \cos \theta \cos \phi \cos(\phi'_0 - 2\phi) \int_0^a J_1\left(\frac{x'_{11} \rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\ & + \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \theta \cos \phi \cos \phi'_0 \int_0^a J'_1\left(\frac{x'_{11} \rho'}{a}\right) J_0(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' - \\ & - \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \theta \cos \phi \cos(\phi'_0 - 2\phi) \int_0^a J'_1\left(\frac{x'_{11} \rho'}{a}\right) J_2(k\rho' \sin \theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' \end{aligned} \quad (12.5.2)$$

Mettendo in evidenza gli integrale comuni, l'espressione (12.5.2) conterrà quattro termini.

$$\begin{aligned}
 & \frac{i\omega\mu}{h^2\lambda^2} A_1\pi \cos\theta [\sin\phi \sin\phi'_0 + \cos\phi \cos\phi'_0] \cdot \\
 & \cdot \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1\pi \cos\theta [\sin\phi \sin\phi'_0 + \cos\phi \cos\phi'_0] \cdot \\
 & \cdot \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' + \\
 & + \frac{i\omega\mu}{h^2\lambda^2} A_1\pi \cos\theta [\cos\phi \cos(\phi'_0 - 2\phi) - \sin\phi \sin(\phi'_0 - 2\phi)] \cdot \\
 & \cdot \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1\pi \cos\theta [\sin\phi \sin(\phi'_0 - 2\phi) - \cos\phi \cos(\phi'_0 - 2\phi)] \cdot \\
 & \cdot \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho'
 \end{aligned} \tag{12.5.3}$$

ossia:

$$\begin{aligned}
 & \frac{i\omega\mu}{h^2\lambda^2} A_1\pi \cos\theta \cos(\phi - \phi'_0) \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) d\rho' + \\
 & + \frac{i\omega\mu}{h\lambda^2} A_1\pi \cos\theta \cos(\phi - \phi'_0) \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_0(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' + \\
 & + \frac{i\omega\mu}{h^2\lambda^2} A_1\pi \cos\theta \cos(\phi'_0 - \phi) \int_0^a J_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho' - \\
 & - \frac{i\omega\mu}{h^2\lambda^2} A_1\pi \cos\theta \cos(\phi'_0 - \phi) \int_0^a J'_1\left(\frac{x'_{11}\rho'}{a}\right) J_2(k\rho' \sin\theta) \exp\left(-\frac{ik\rho'^2}{2l}\right) \rho' d\rho'
 \end{aligned} \tag{12.5.4}$$

e, che si può ancora scrivere:

$$\begin{aligned}
 & \frac{i\omega\mu}{h\lambda^2} A_1 \pi \cos \theta \cos(\phi - \phi'_0) \left[\frac{1}{h} \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' + \right. \\
 & \left. + \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_0(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' \right] + \\
 & \left[\frac{1}{h} \int_0^a J_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) d\rho' - \right. \\
 & \left. - \int_0^a J'_1 \left(\frac{x'_{11}\rho'}{a} \right) J_2(k\rho' \sin \theta) \exp \left(-\frac{ik\rho'^2}{2l} \right) \rho' d\rho' \right]
 \end{aligned} \tag{12.5.5}$$

In definitiva i campi elettrici sono:

$$E_\theta = -A_1 \frac{e^{-ikr}}{r} \frac{\mu\omega}{\lambda h} (\text{Superficie apertura}) \sin(\phi - \phi'_0) [W_0 - W_2] \tag{12.5.6}$$

$$E_\phi = -A_1 \frac{e^{-ikr}}{r} \frac{\mu\omega}{\lambda h} (\text{Superficie apertura}) \cos \theta \cos(\phi - \phi'_0) [W_0 + W_2] \tag{12.5.7}$$

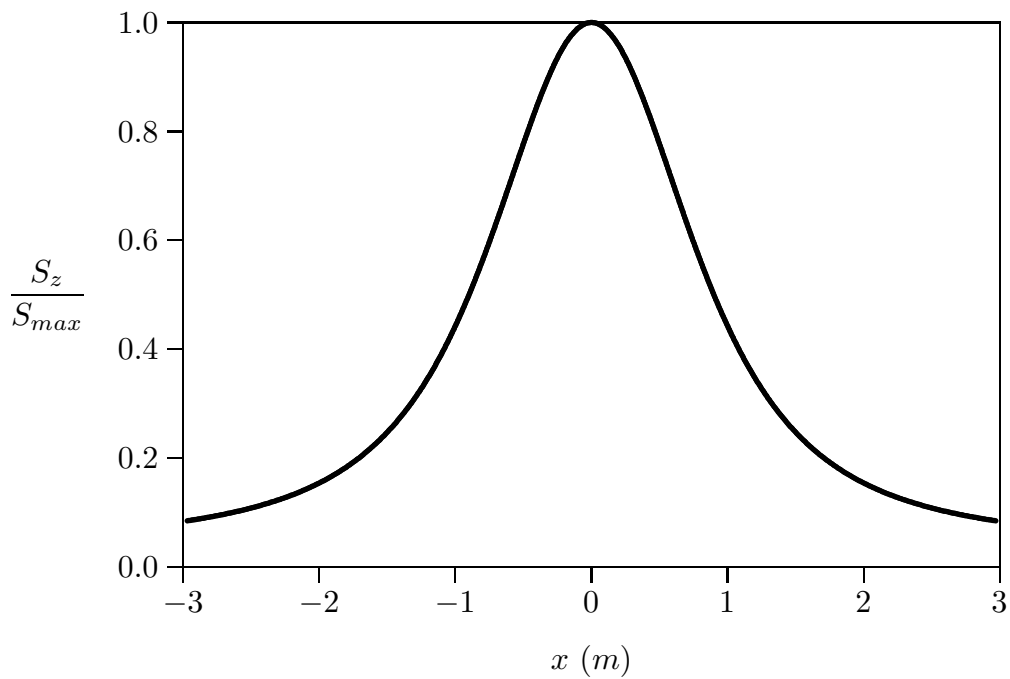
dove:

$$W_0 = \int_0^1 w J_0(x'_{11}w) J_0(kaw \sin \theta) \exp \left(-\frac{ika^2w^2}{2l} \right) dw \tag{12.5.8}$$

$$W_2 = \int_0^1 w J_2(x'_{11}w) J_2(kaw \sin \theta) \exp \left(-\frac{ika^2w^2}{2l} \right) dw \tag{12.5.9}$$

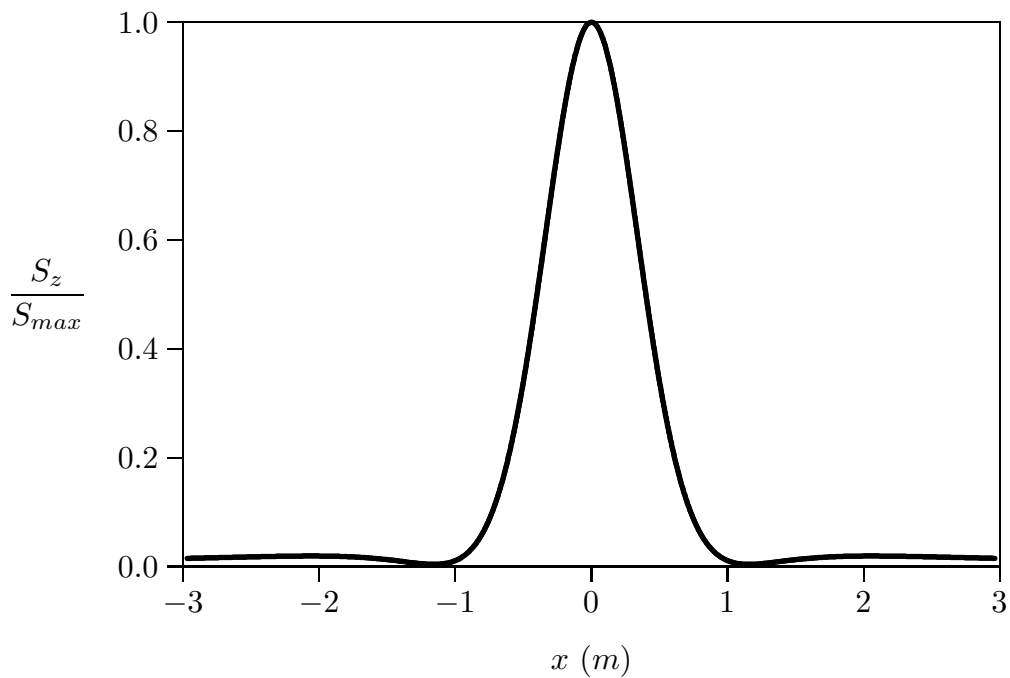
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 0.5\lambda$, $l = 7\lambda$, $z = 50\lambda$



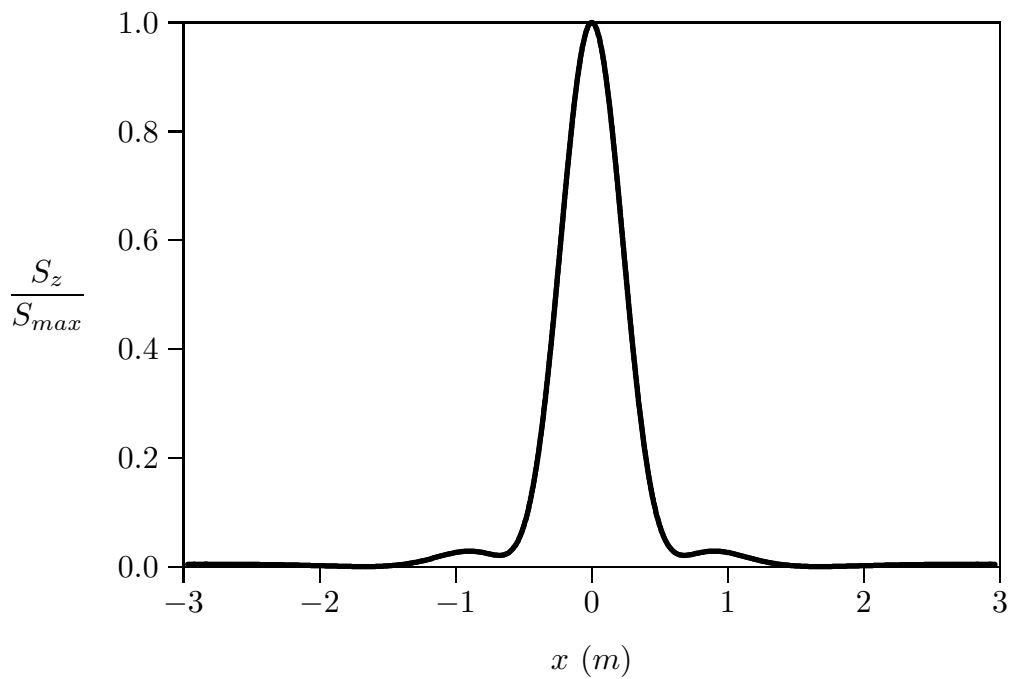
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = \lambda$, $l = 7\lambda$, $z = 50\lambda$



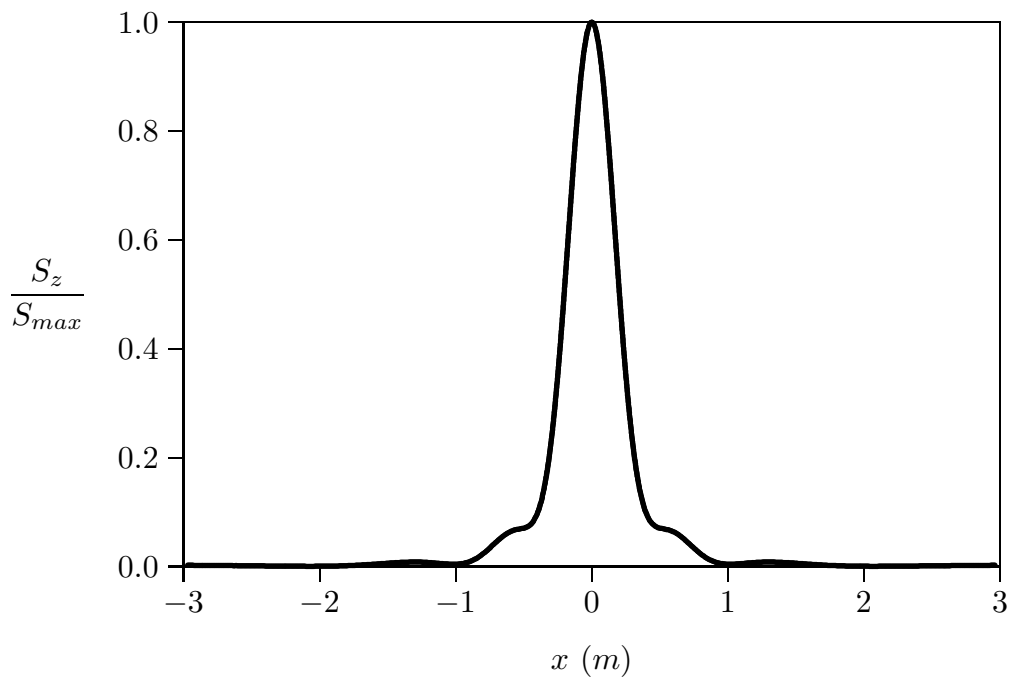
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 1.5\lambda$, $l = 7\lambda$, $z = 50\lambda$



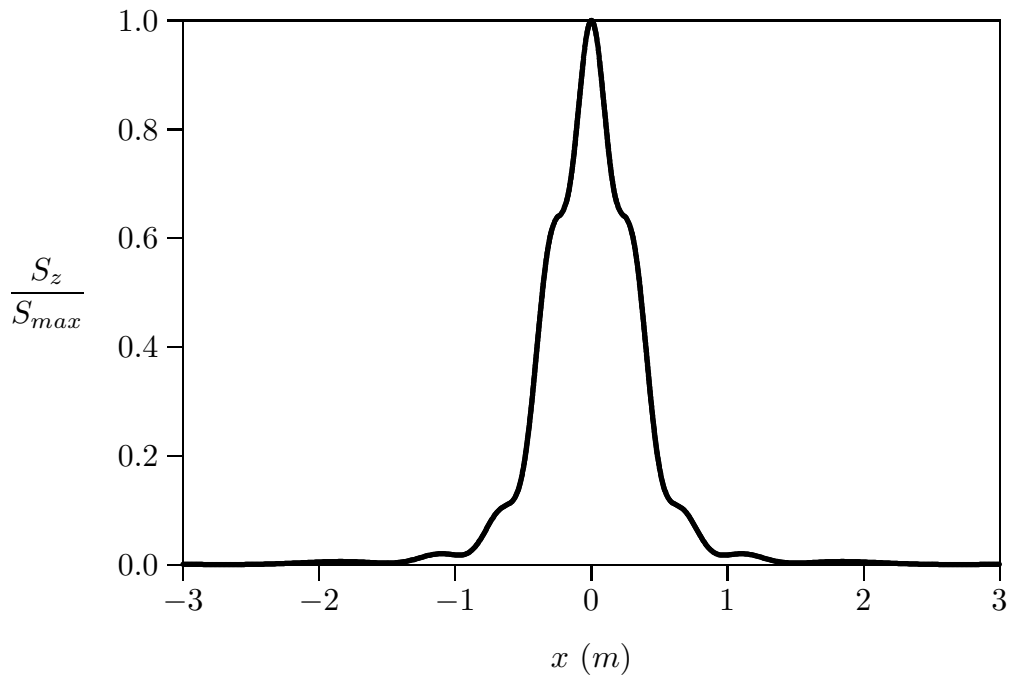
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 2\lambda$, $l = 7\lambda$, $z = 50\lambda$



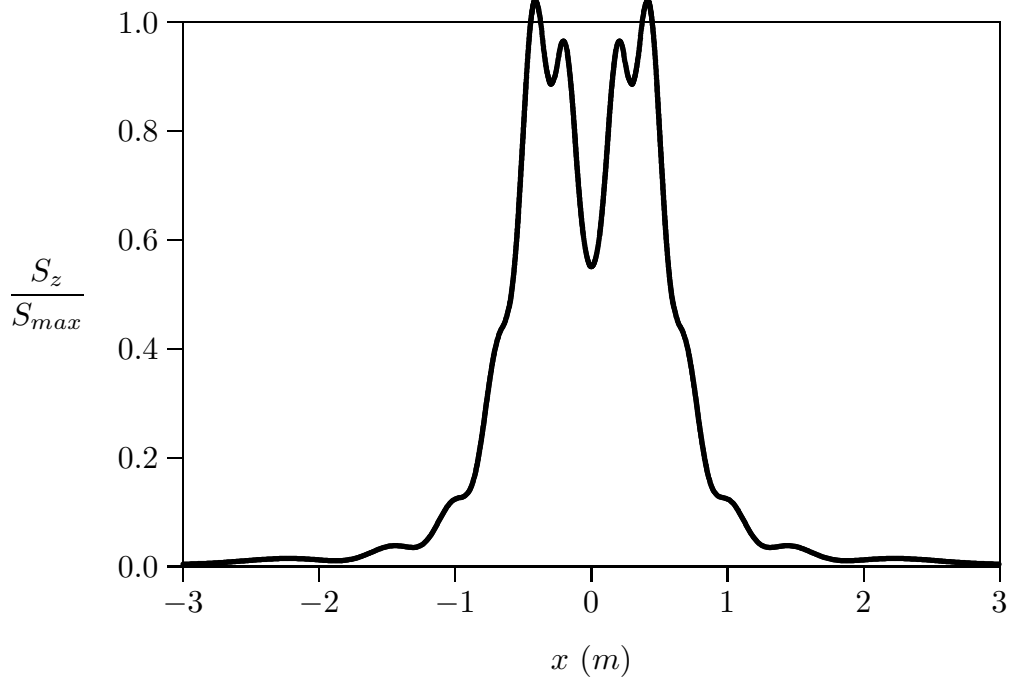
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 3\lambda$, $l = 7\lambda$, $z = 50\lambda$



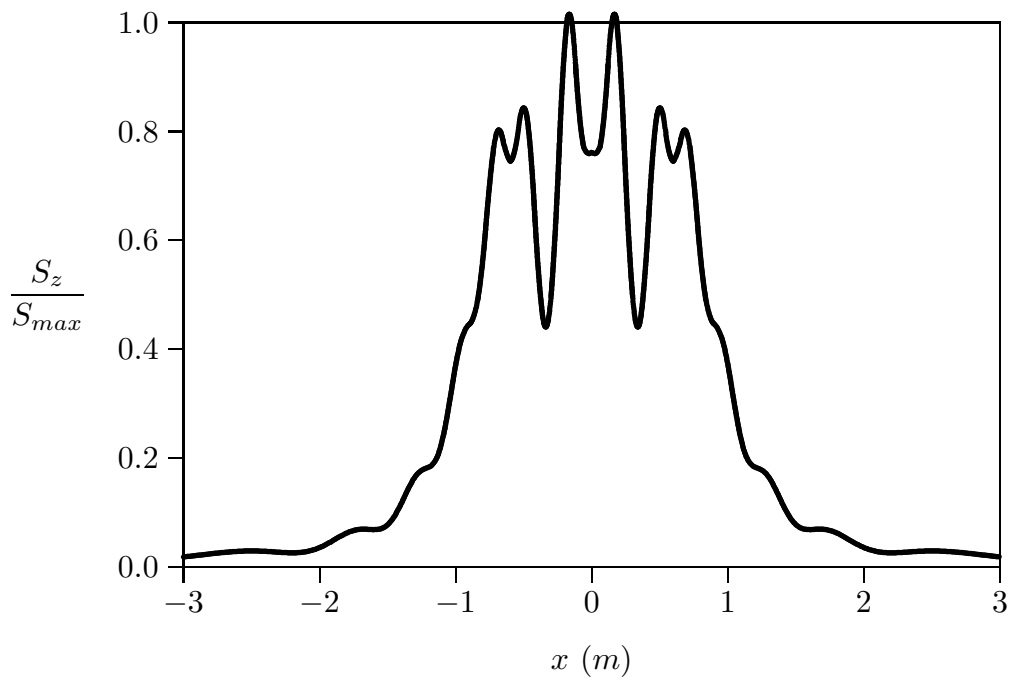
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 4\lambda$, $l = 7\lambda$, $z = 50\lambda$



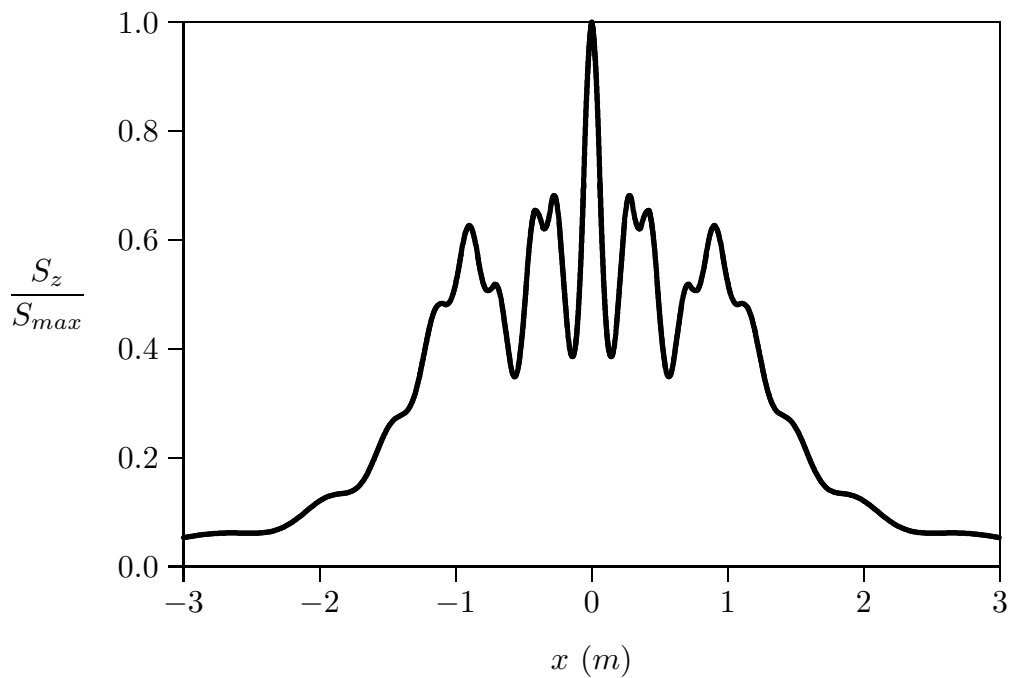
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 5\lambda$, $l = 7\lambda$, $z = 50\lambda$



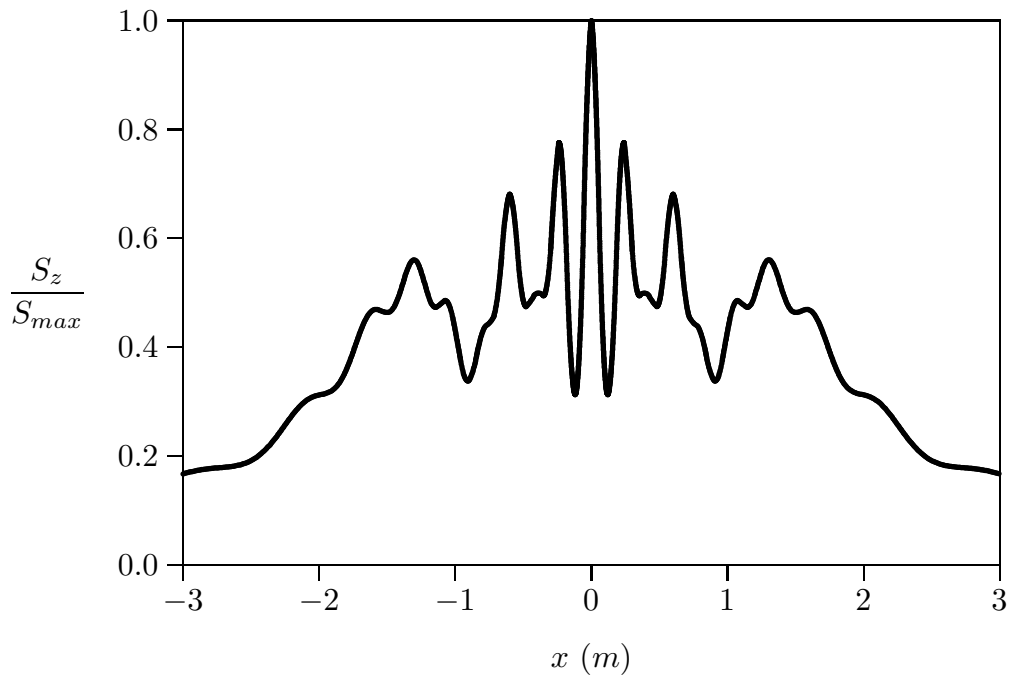
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 6\lambda$, $l = 7\lambda$, $z = 50\lambda$



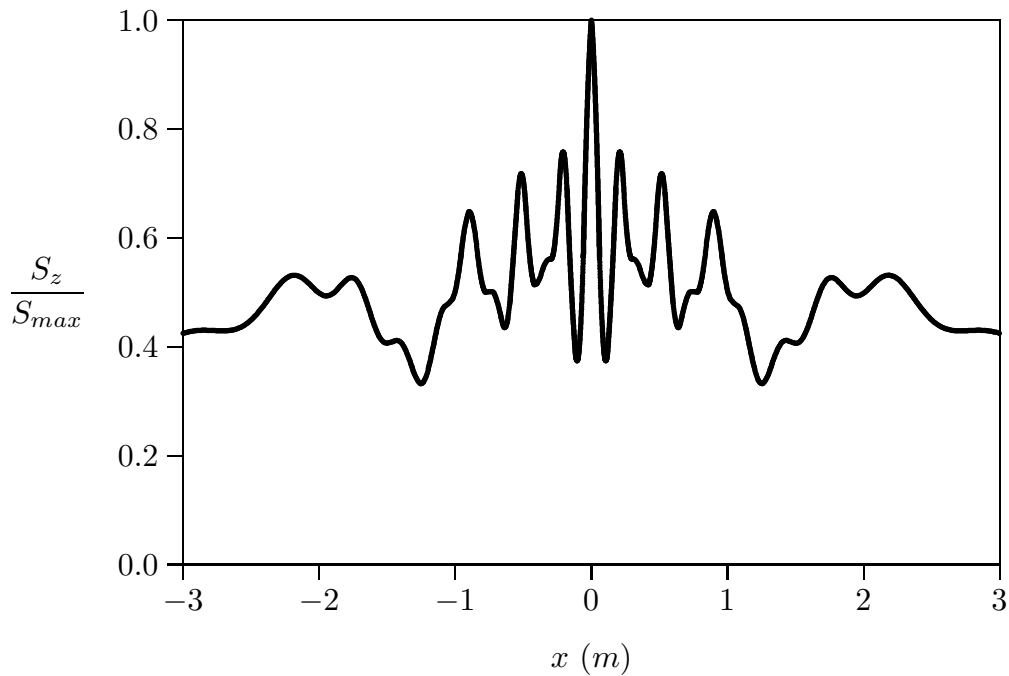
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 7\lambda$, $l = 7\lambda$, $z = 50\lambda$



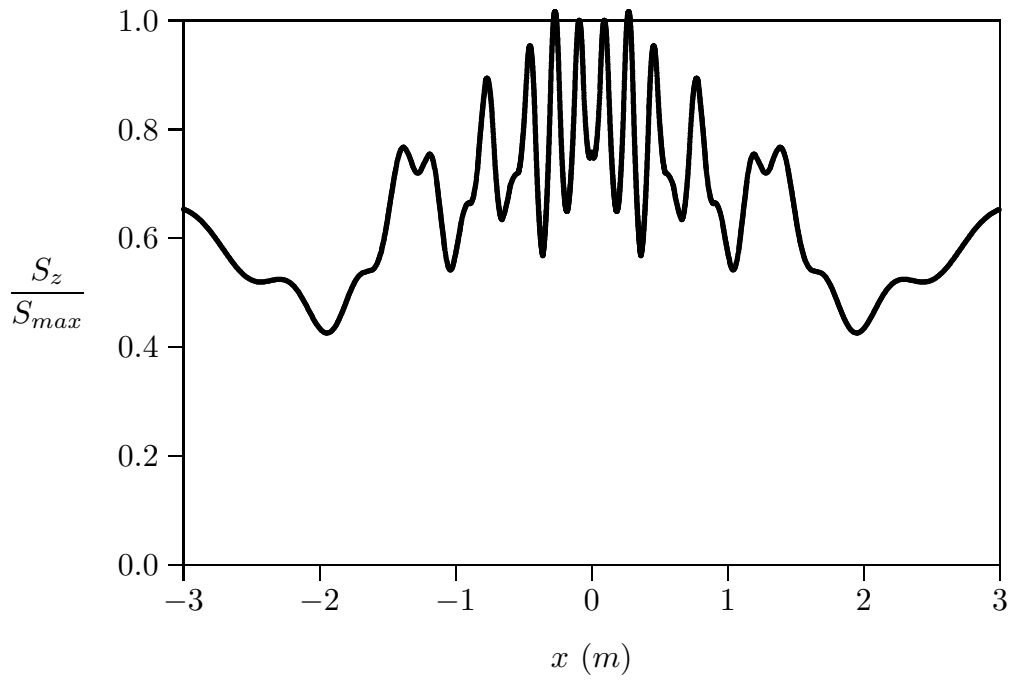
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 8\lambda$, $l = 7\lambda$, $z = 50\lambda$



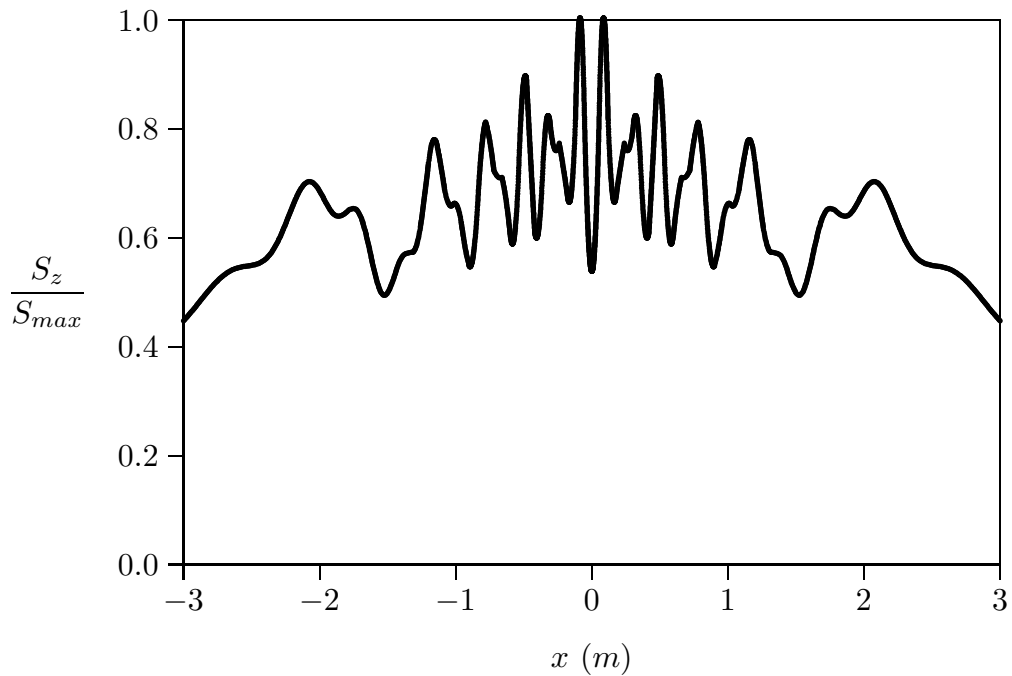
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 9\lambda$, $l = 7\lambda$, $z = 50\lambda$



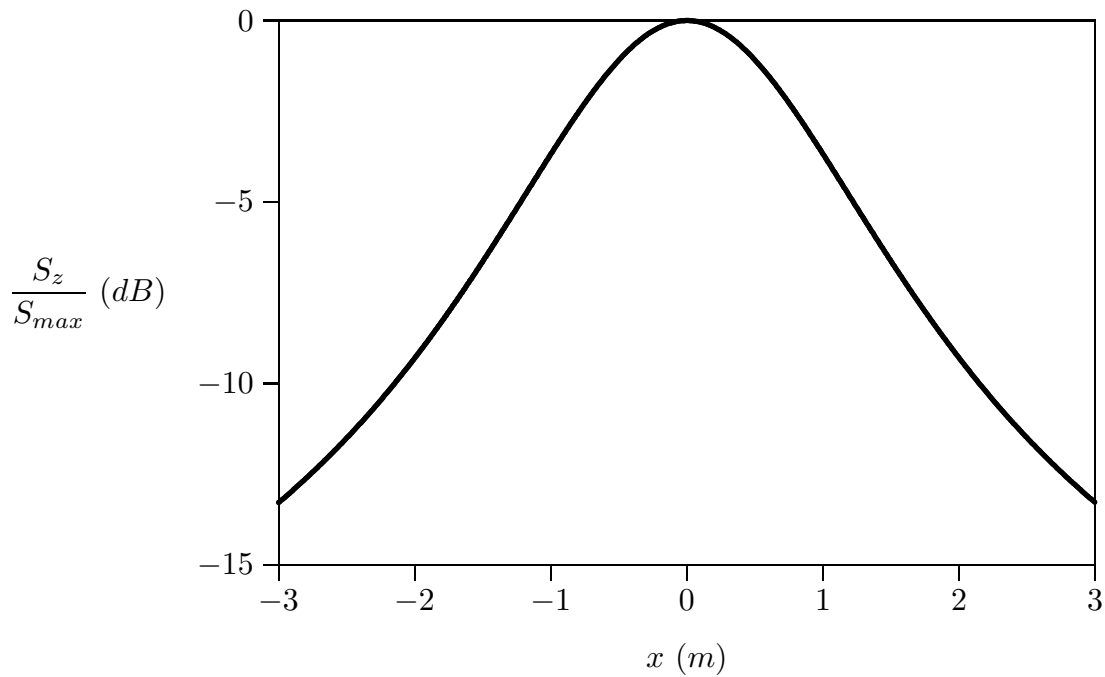
Diffrazione di Fraunhofer dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 10\lambda$, $l = 7\lambda$, $z = 50\lambda$



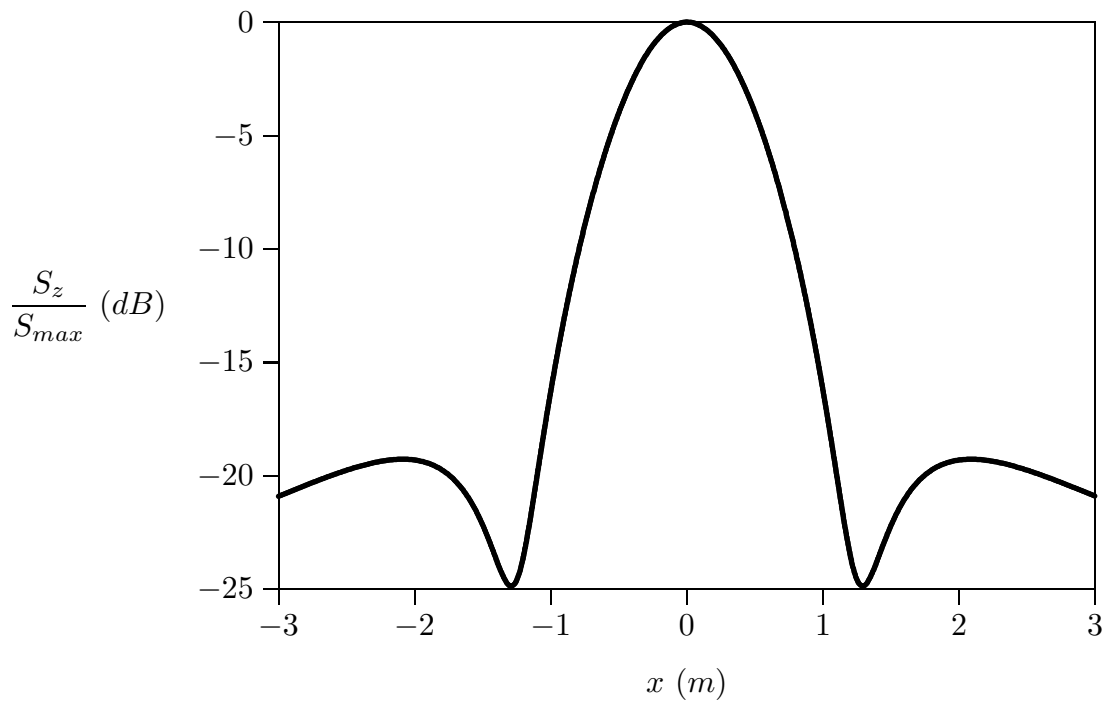
Diffrazione di Fraunhofer dell'horn circolare nel piano xy in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 0.5\lambda$, $l = 7\lambda$, $z = 50\lambda$



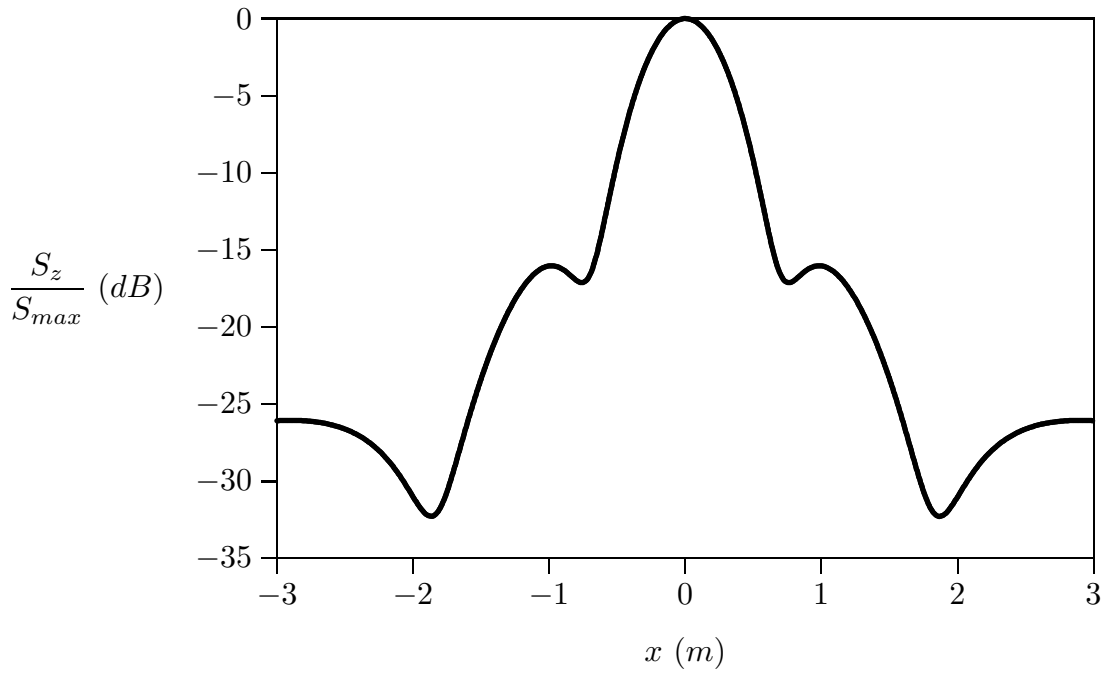
Diffrazione di Fraunhofer dell'horn circolare nel piano xy in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = \lambda$, $l = 7\lambda$, $z = 50\lambda$



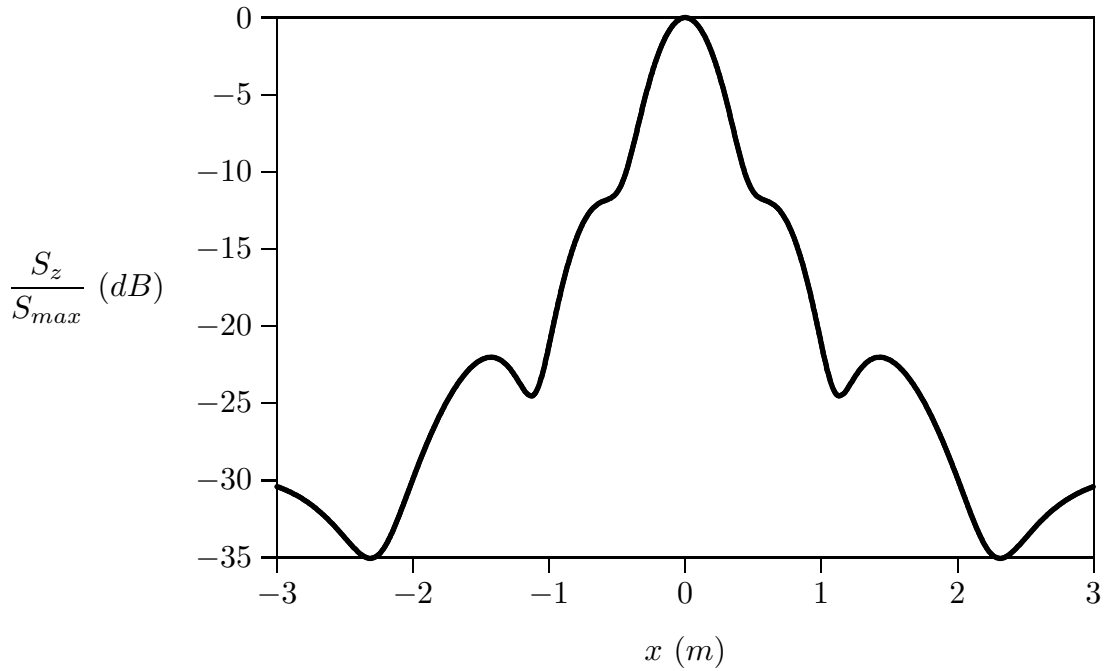
Diffrazione di Fraunhofer dell'horn circolare nel piano xy in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 1.5\lambda$, $l = 7\lambda$, $z = 50\lambda$



Diffrazione di Fraunhofer dell'horn circolare nel piano xy in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 2\lambda$, $l = 7\lambda$, $z = 50\lambda$



Programma Matlab: **'horncircolare.m'**

```
01 %%% Diffrazione da horn circolare %%%
02 delete(get(0,'children'));
03 freq=10e9;%%frequenza della radiazione emessa%%
04 c=3e8;
05 lambda=c./freq;%%lunghezza d'onda%%
06 z=50.*lambda;%%distanza dall'horn %%
07 a=1.*lambda;%%raggio dell'horn%%
08 l=7.*lambda;%%lunghezza dell'horn%%
09 xp11=1.841;
10 k=(2.*pi)./lambda;
11 x=-99.*lambda:0.5.*lambda:+99.*lambda;
12 y=0;
13 r=sqrt(x.^2+y.^2+z.^2);
14 s=(sqrt(x.^2+y.^2))./r;%%sintheta%%
15 ct=z./r;%%costheta%%
16 %%% Integrazione %%%
17 Ris=1000; %%precisione di integrazione%%
18 wmax=1;
19 for k1=1:Ris+1
20 w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 end
22 W0=0;
23 W2=0;
24 for j = 2:Ris+1
25 W0=W0+w(j).*bessel(0,w(j).*xp11).*bessel(0,k.*a.*w(j).*s)...
26 .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 W2=W2+w(j).*bessel(2,w(j).*xp11).*bessel(2,k.*a.*w(j).*s)...
28 .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
29 end
30 Poynting=(abs(W0-W2)).^2;
31 Poynting_z=(abs(W0-W2)).^2.*ct;
32 Poyntingmax=max(Poynting_z);
33 plot(x,Poynting./Poyntingmax)
34 %%% Istruzioni per trasferire i dati in formato TEX%%
35 B=[x;Poynting./Poyntingmax];
36 fid=fopen('pippo.tex','w');
37 fprintf(fid,'\n');
38 fprintf(fid,'%5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f\n',B');
39 fclose(fid);
```

Se si vuole graficare il diagramma di radiazione proiettato nel piano parallelo al piano xy **ma espresso in dB** basta sostituire al programma precedente la istruzioni 33 ed eventualmente la 35, come nel seguente programma.

Programma Matlab: **'horncircolaredb.m'**

```

01 %%%%% Diffrazione da horn circolare %%%%%
02 delete(get(0,'children'));
03 freq=10e9; %%%%% frequenza della radiazione emessa %%%%%
04 c=3e8;
05 lambda=c./freq; %%%%% lunghezza d'onda %%%%%
06 z=50.*lambda; %%%%% distanza dall'horn %%%%%
07 a=1.*lambda; %%%%% raggio dell'horn %%%%%
08 l=7.*lambda; %%%%% lunghezza dell'horn %%%%%
09 xp11=1.841;
10 k=(2.*pi)./lambda;
11 x=-99.*lambda:0.5.*lambda:+99.*lambda;
12 y=0;
13 r=sqrt(x.^2+y.^2+z.^2);
14 s=(sqrt(x.^2+y.^2))./r; %%%%% sintheta %%%%%
15 ct=z./r; %%%%% costheta %%%%%
16 %%%%% Integrazione %%%%%
17 Ris=1000; %%%%% precisione di integrazione %%%%%
18 wmax=1;
19 for k1=1:Ris+1
20 w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 end
22 W0=0;
23 W2=0;
24 for j = 2:Ris+1
25 W0=W0+w(j).*bessel(0,w(j).*xp11).*bessel(0,k.*a.*w(j).*s)...
26 .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 W2=W2+w(j).*bessel(2,w(j).*xp11).*bessel(2,k.*a.*w(j).*s)...
28 .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
29 end
30 Poynting=(abs(W0-W2)).^2;
31 Poynting_z=(abs(W0-W2)).^2.*ct;
32 Poyntingmax=max(Poynting_z);
33 plot(x,10.*log10(Poynting./Poyntingmax))
34 %%%%% Istruzioni per trasferire i dati in formato TEX %%%%%
35 B=[x;10.*log10(Poynting_z./Poyntingmax)];
36 fid=fopen('pippo.tex','w');
37 fprintf(fid,'\n');
38 fprintf(fid,'%5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f %5.4f\n',B);
39 fclose(fid);

```

Diagramma di radiazione dell'horn circolare

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 0.5\lambda$, $l = 7\lambda$, $z = 50\lambda$

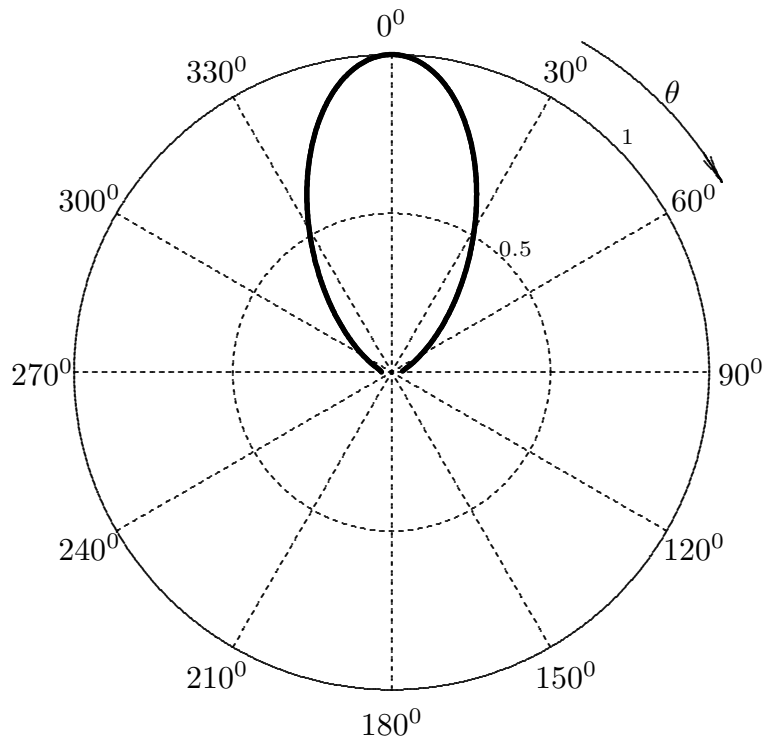


Diagramma di radiazione dell'horn circolare

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 1\lambda$, $l = 7\lambda$, $z = 50\lambda$

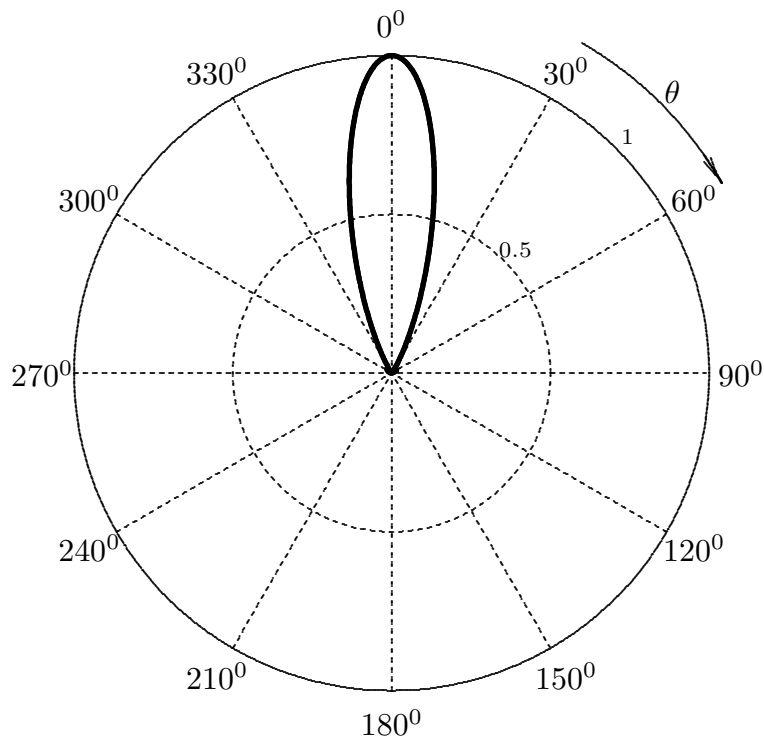


Diagramma di radiazione dell'horn circolare
 $\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 1.5\lambda$, $l = 7\lambda$, $z = 50\lambda$

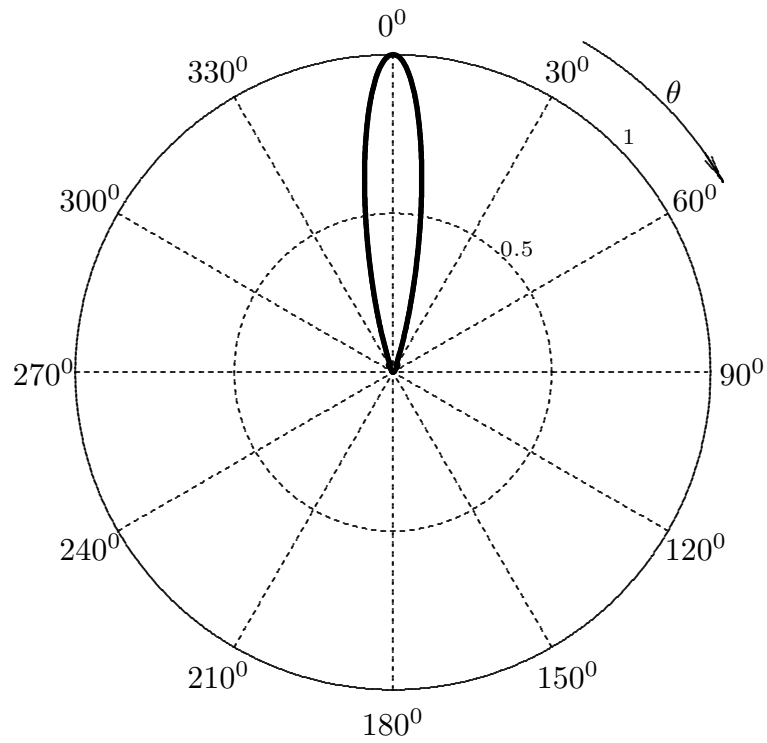


Diagramma di radiazione dell'horn circolare
 $\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 2\lambda$, $l = 7\lambda$, $z = 50\lambda$

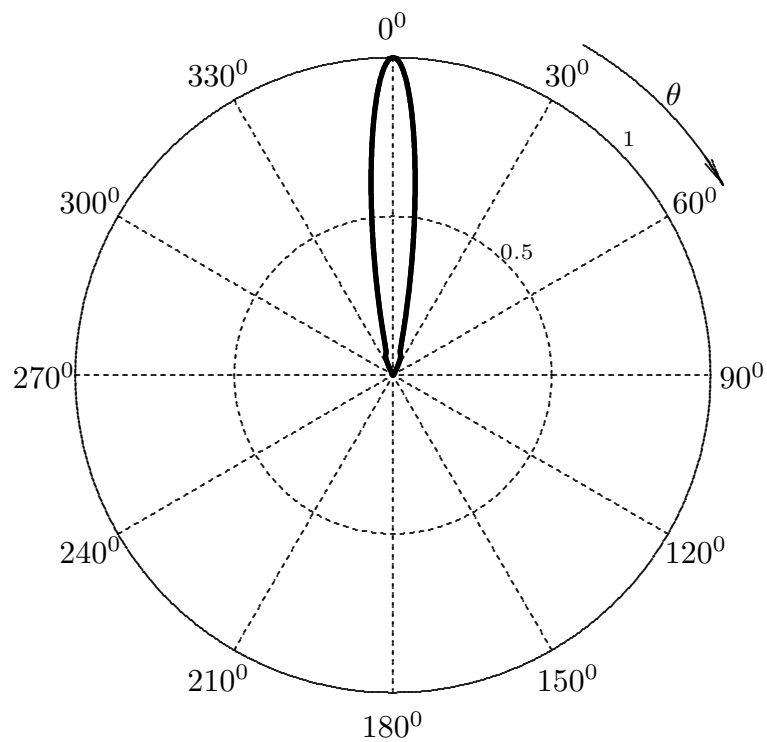


Diagramma di radiazione dell'horn circolare espresso in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 0.5\lambda$, $l = 7\lambda$, $z = 50\lambda$

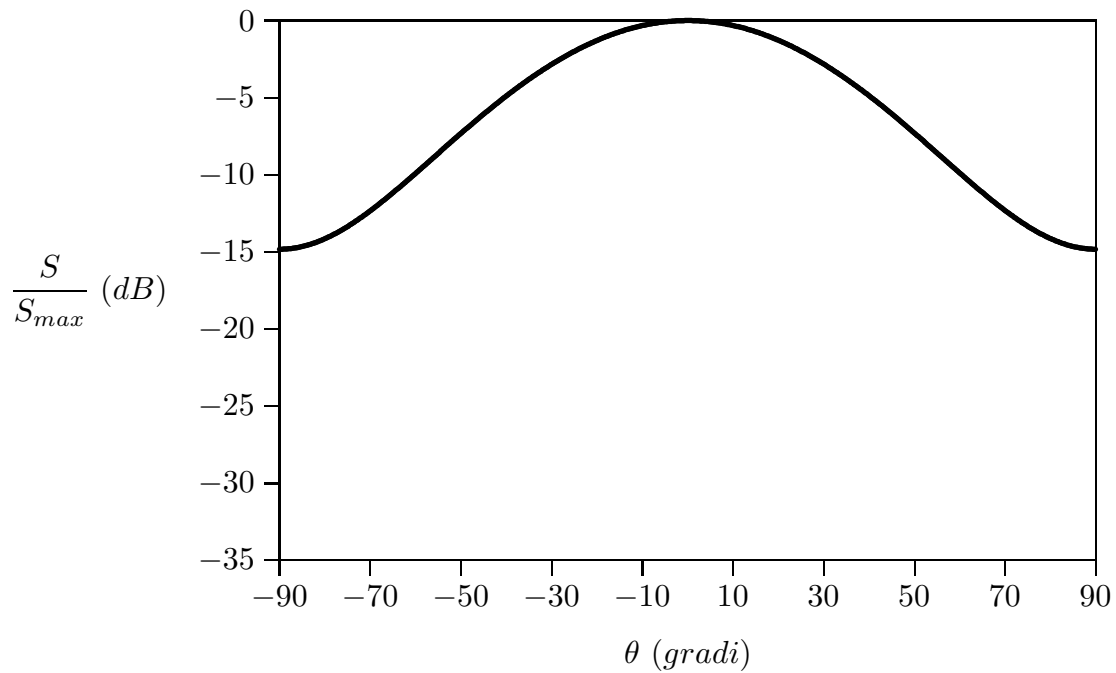


Diagramma di radiazione dell'horn circolare espresso in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = \lambda$, $l = 7\lambda$, $z = 50\lambda$

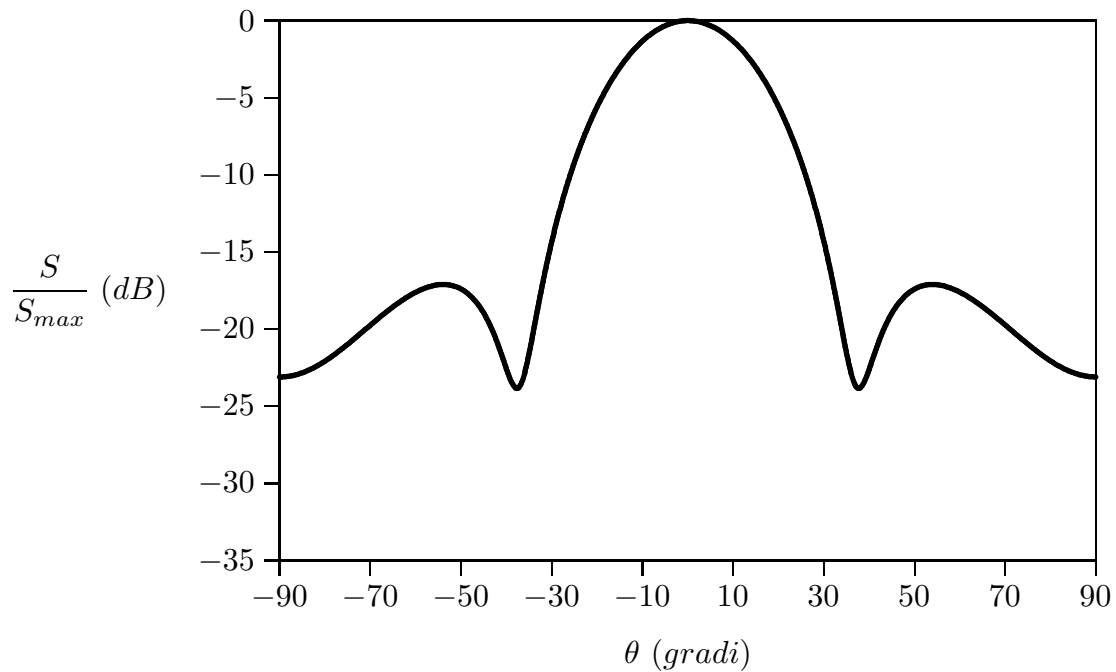


Diagramma di radiazione dell'horn circolare espresso in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 1.5\lambda$, $l = 7\lambda$, $z = 50\lambda$

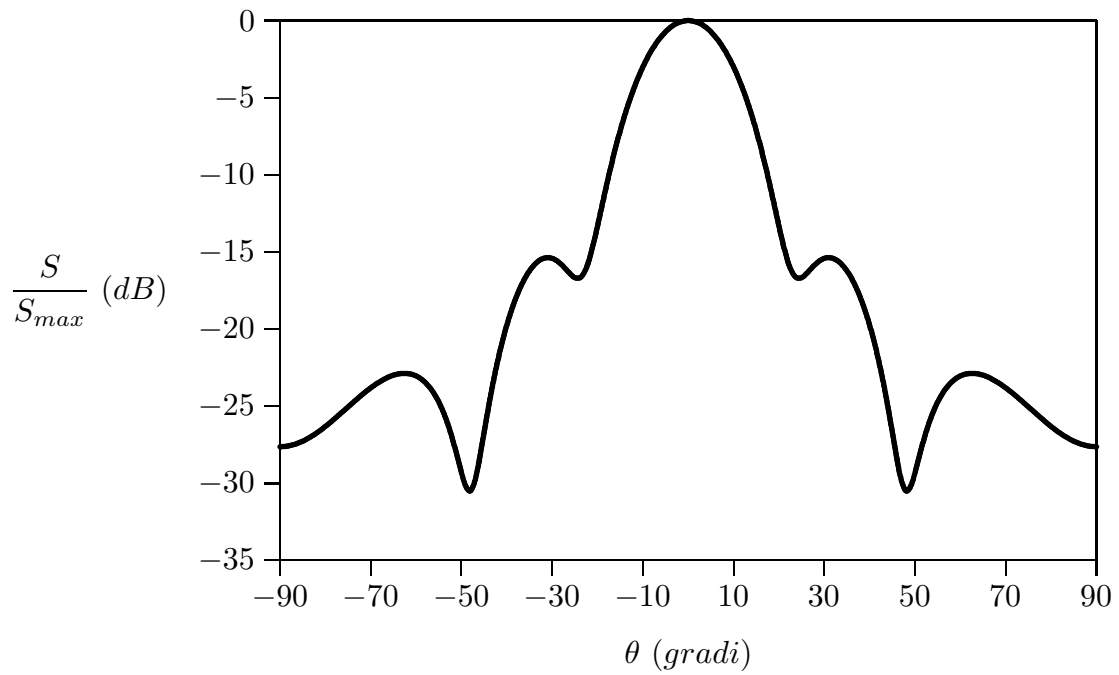
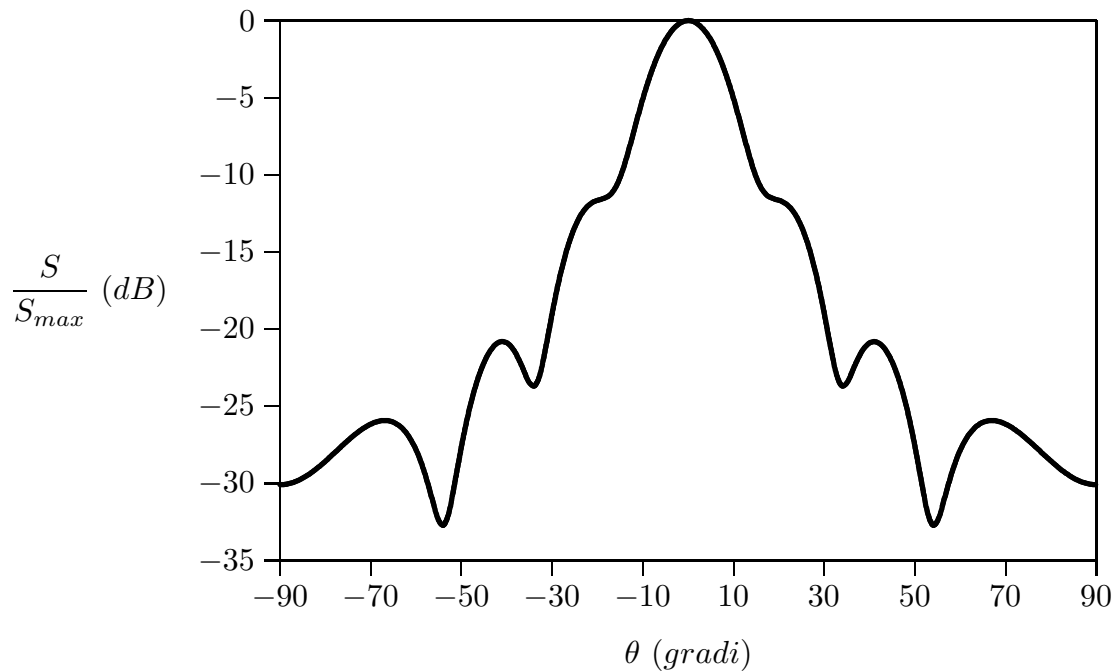


Diagramma di radiazione dell'horn circolare espresso in dB

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 2\lambda$, $l = 7\lambda$, $z = 50\lambda$



Diagrammi di radiazione 3D
Horn circolare: Diffrazione di Fraunhofer ($z=50\lambda$)

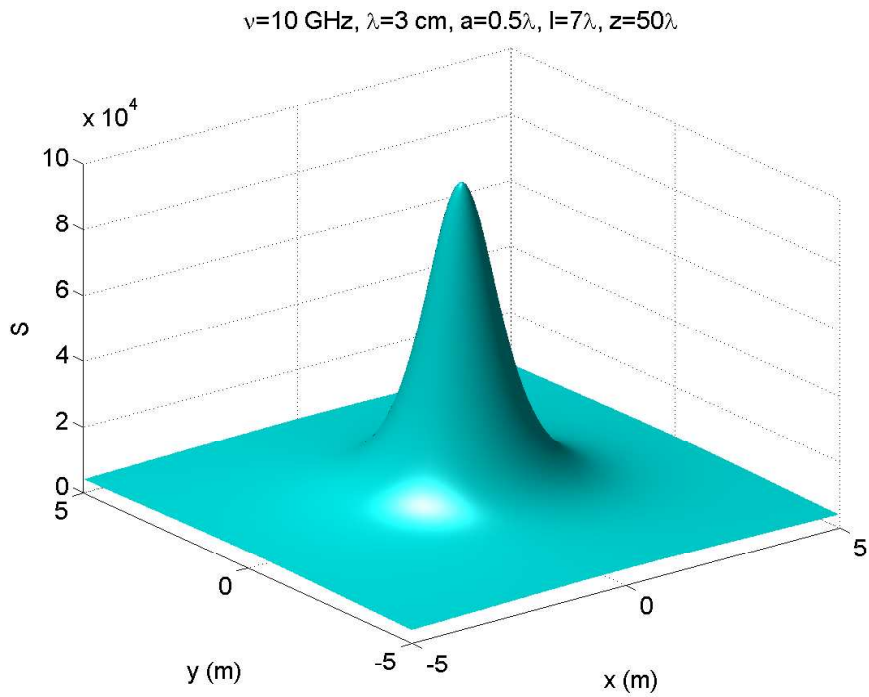


fig.12.5-1

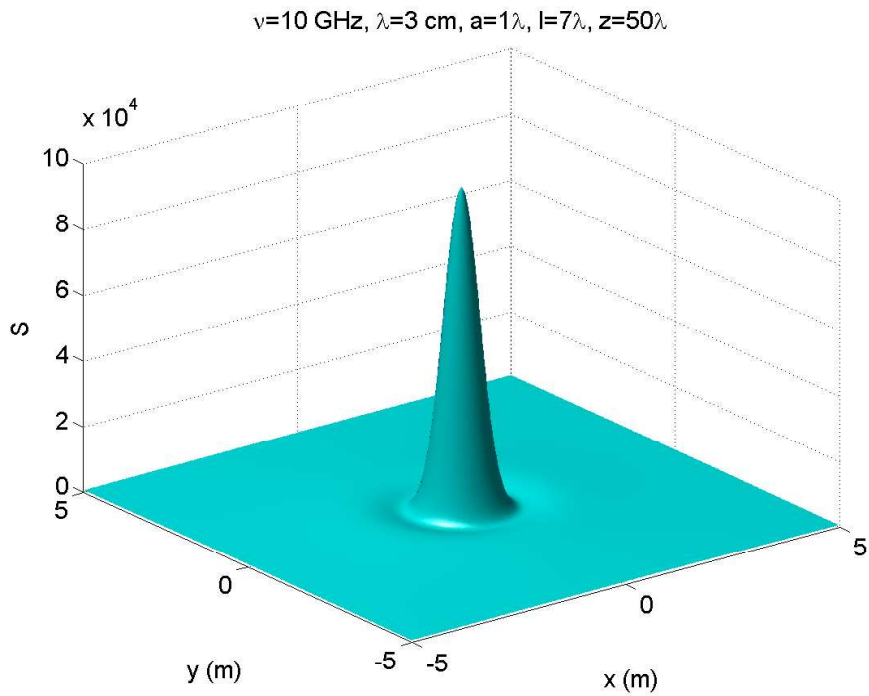


fig.12.5-2

$\nu=10$ GHz, $\lambda=3$ cm, $a=1.5\lambda$, $l=7\lambda$, $z=50\lambda$

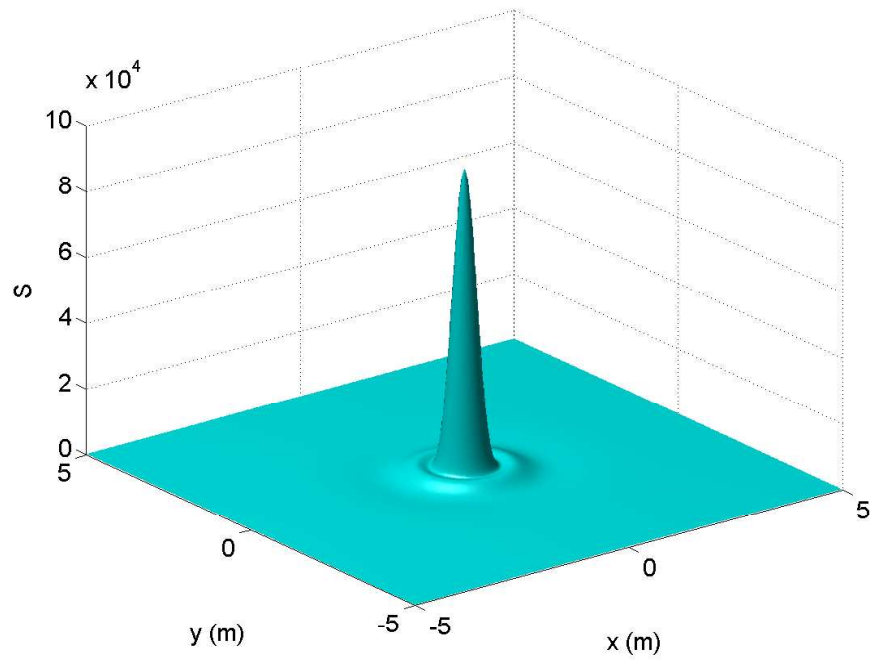


fig.12.5-3

$\nu=10$ GHz, $\lambda=3$ cm, $a=2\lambda$, $l=7\lambda$, $z=50\lambda$

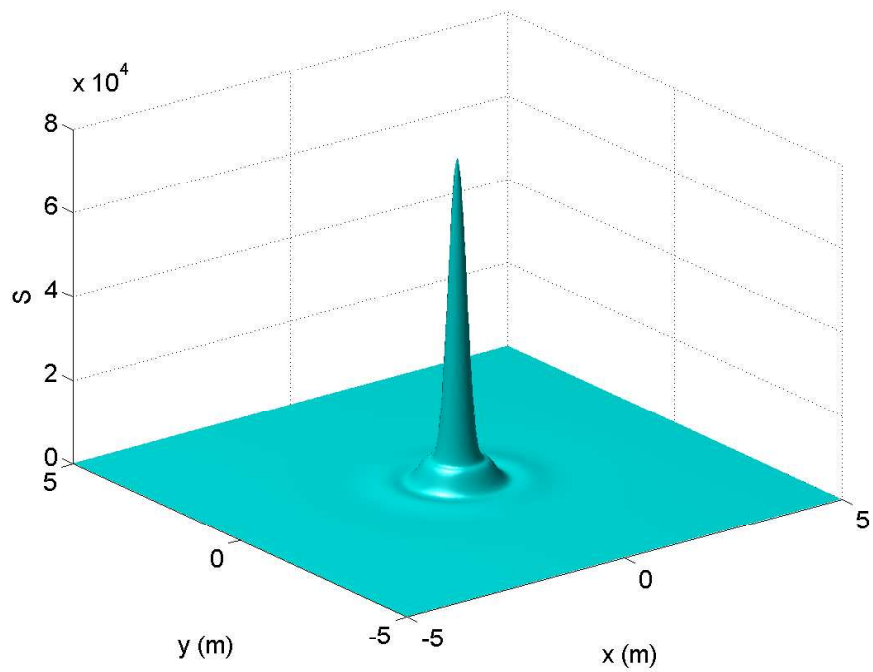


fig.12.5-4

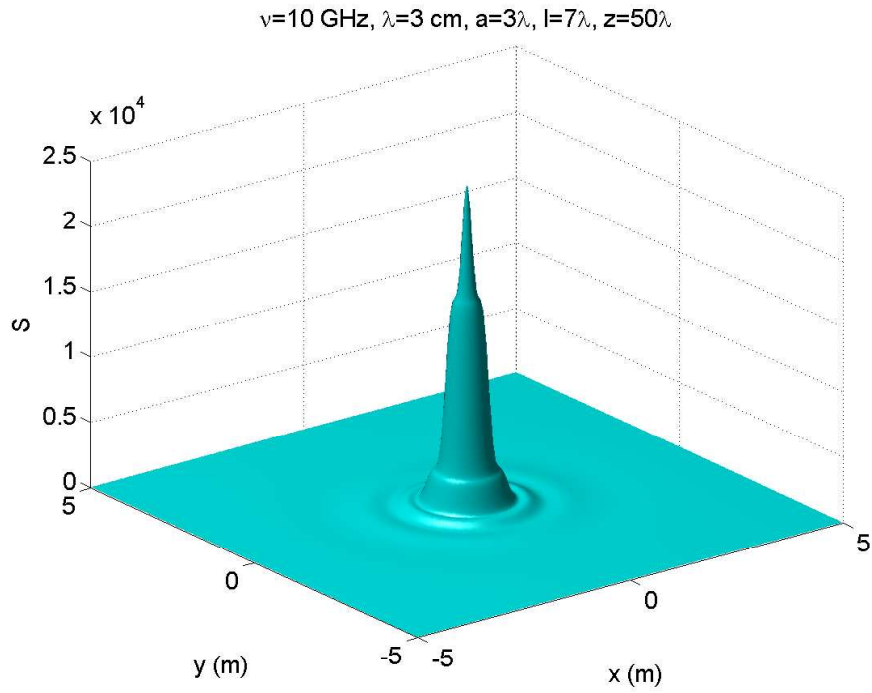


fig.12.5-5

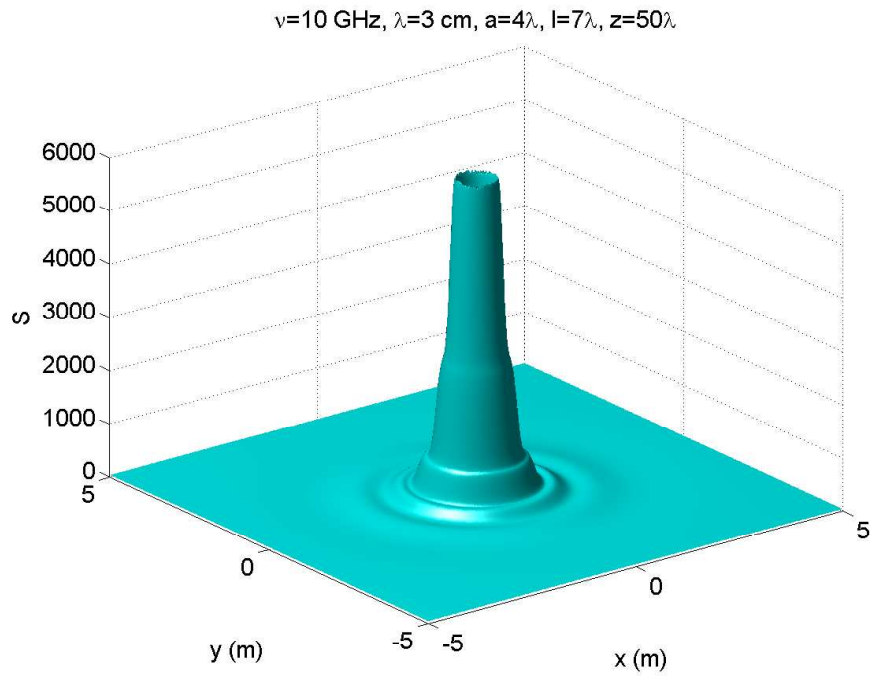


fig.12.5-6

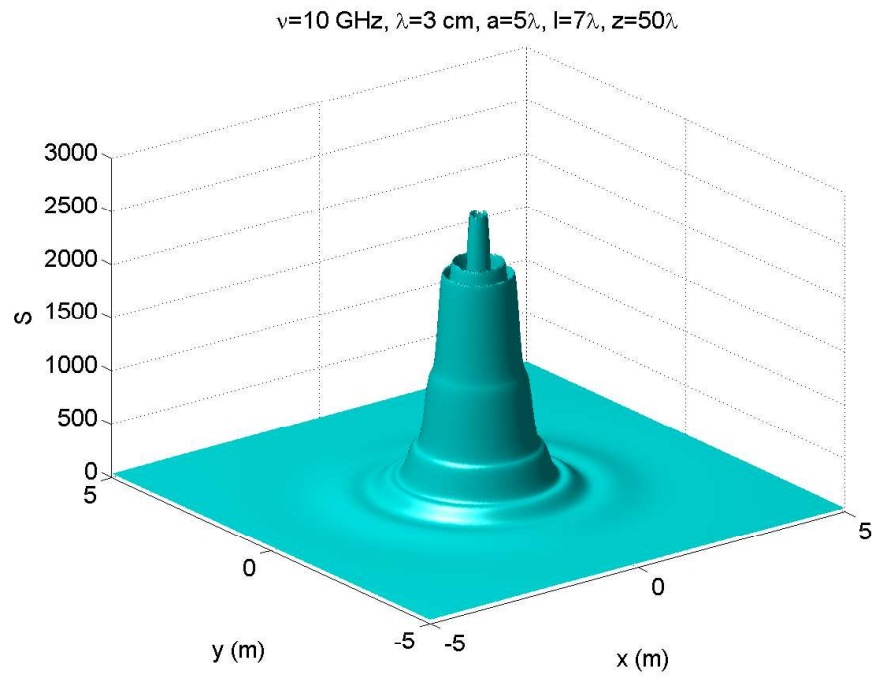


fig.12.5-7

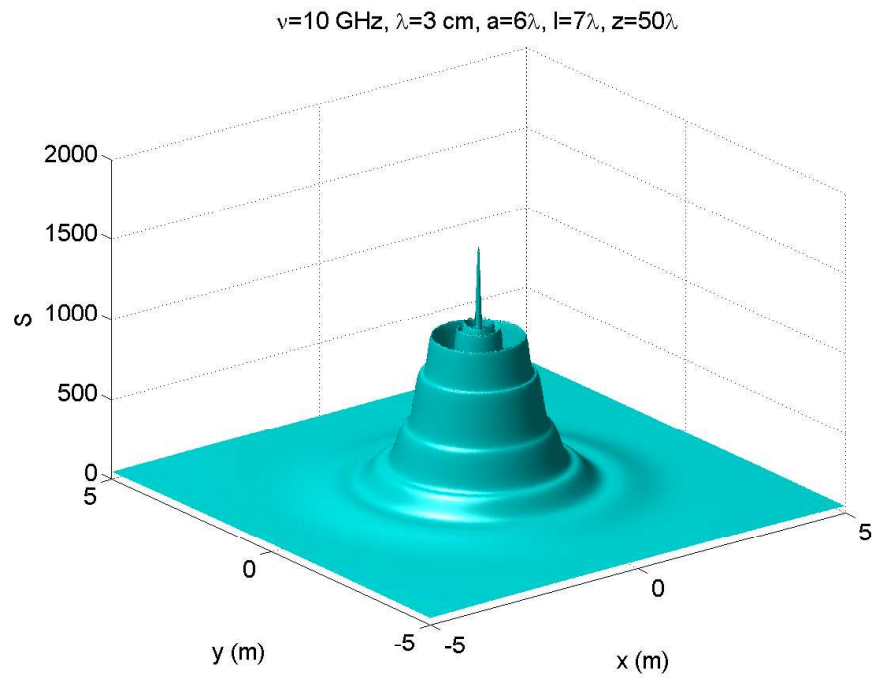


fig.12.5-8

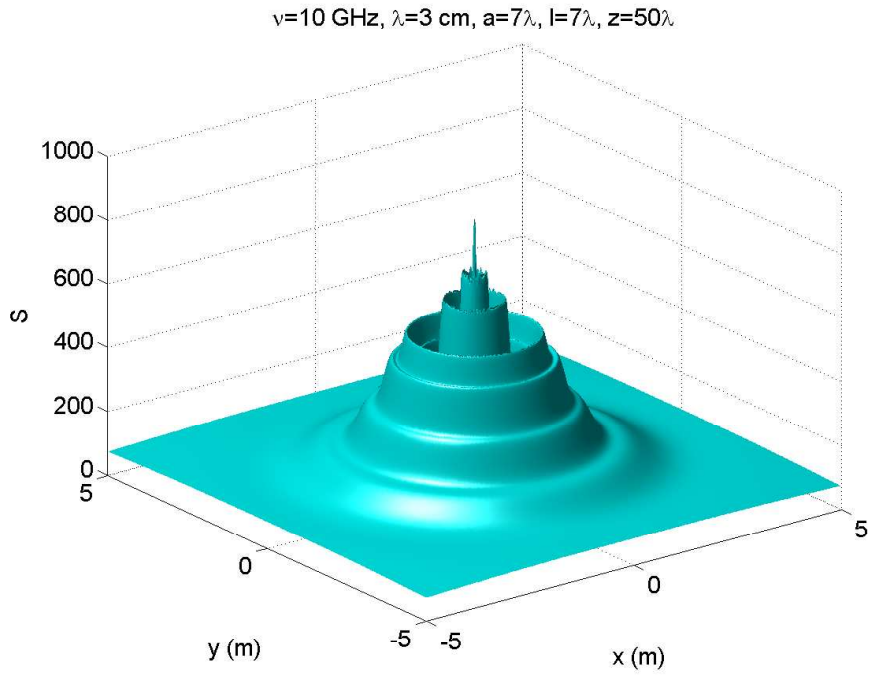


fig.12.5-9

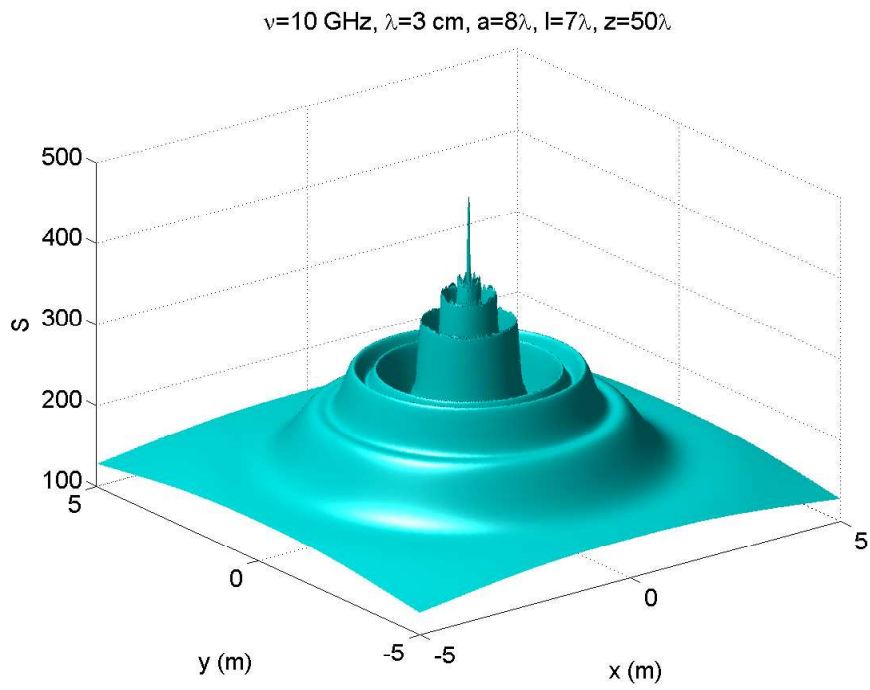


fig.12.5-10

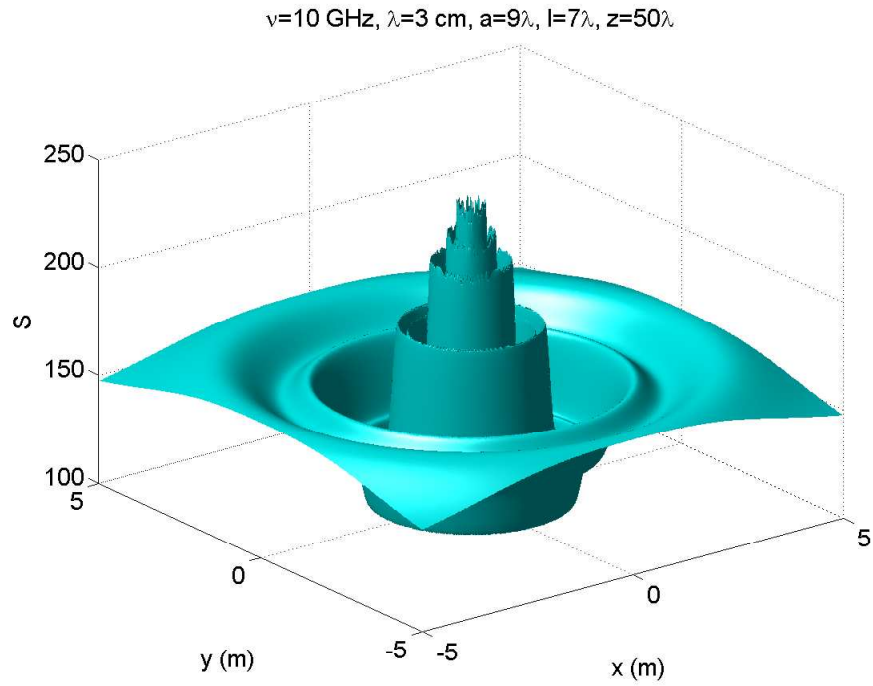


fig.12.5-11

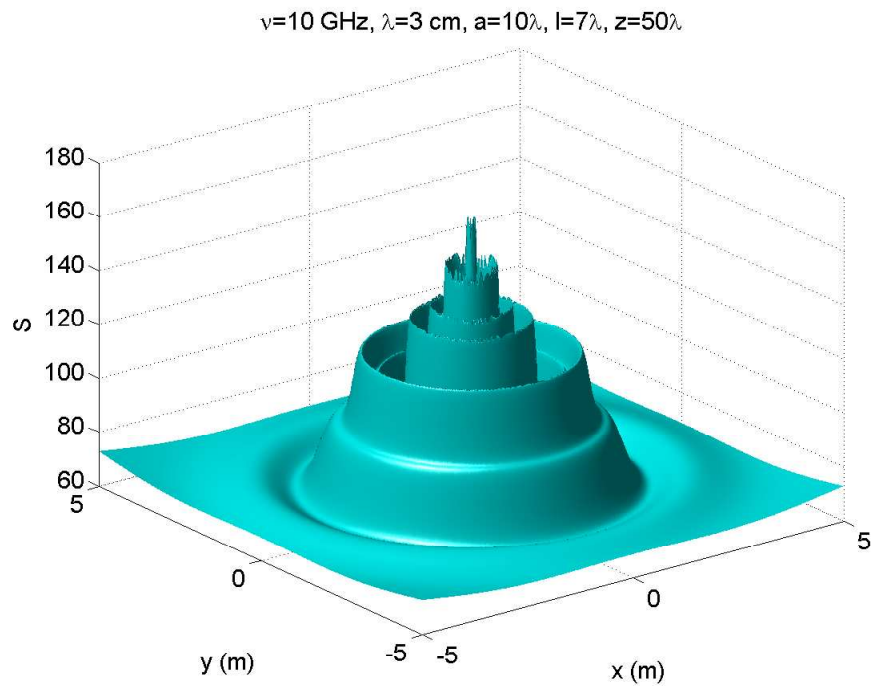


fig.12.5-12

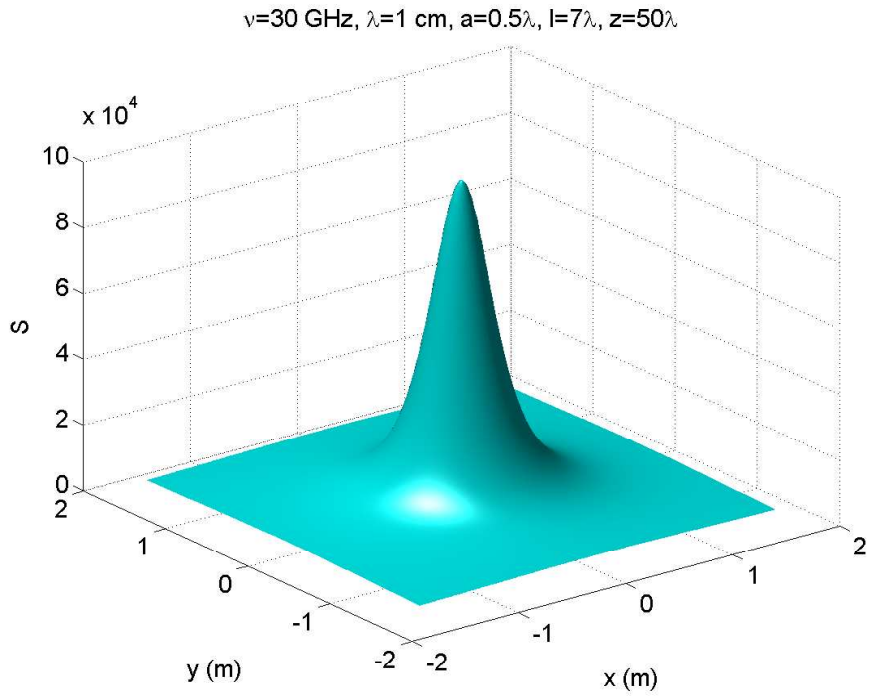


fig.12.5-13

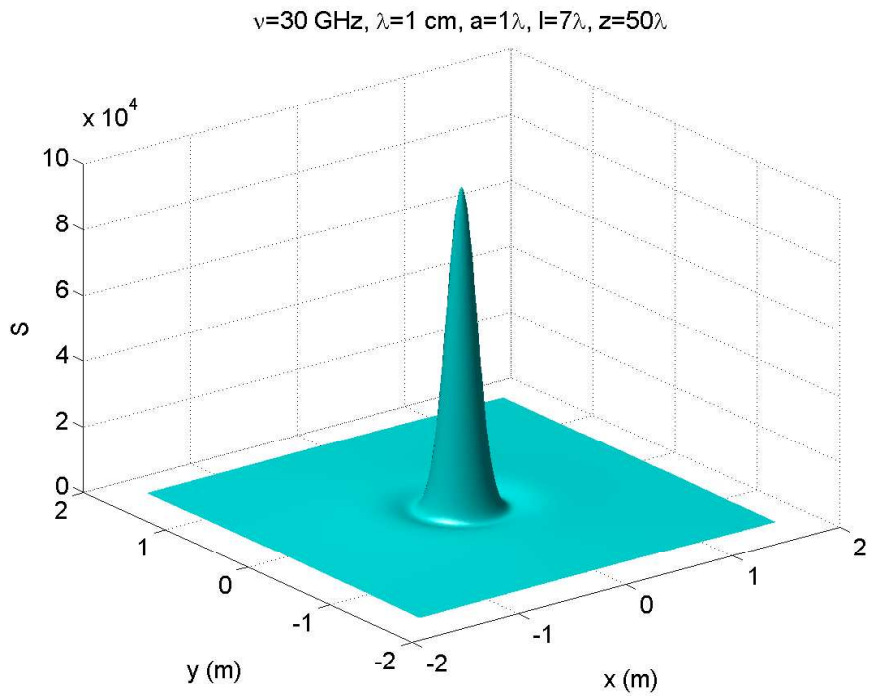


fig.12.5-14

$\nu=30$ GHz, $\lambda=1.5$ cm, $a=1.5\lambda$, $l=7\lambda$, $z=50\lambda$

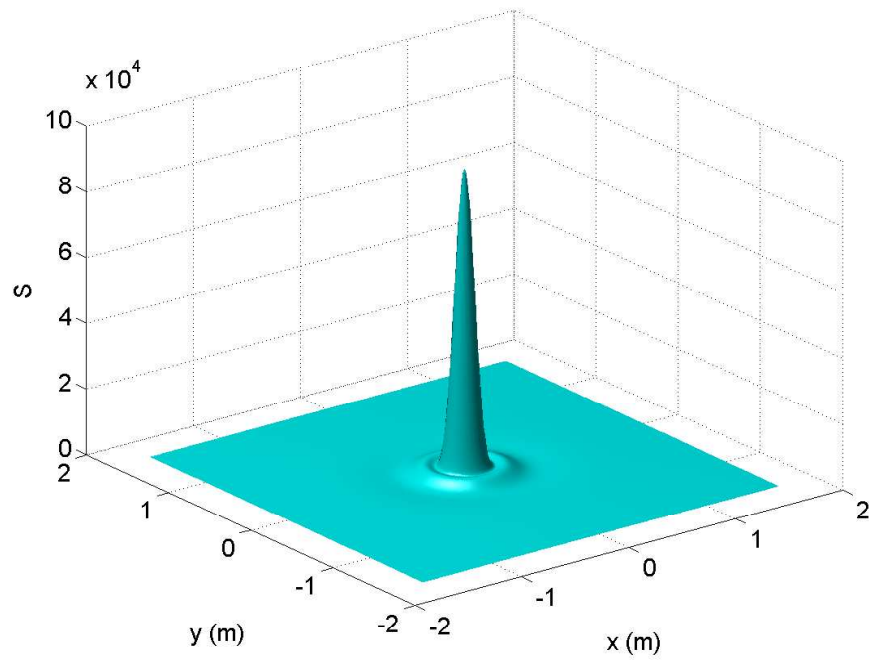


fig.12.5-15

$\nu=30$ GHz, $\lambda=1$ cm, $a=2\lambda$, $l=7\lambda$, $z=50\lambda$

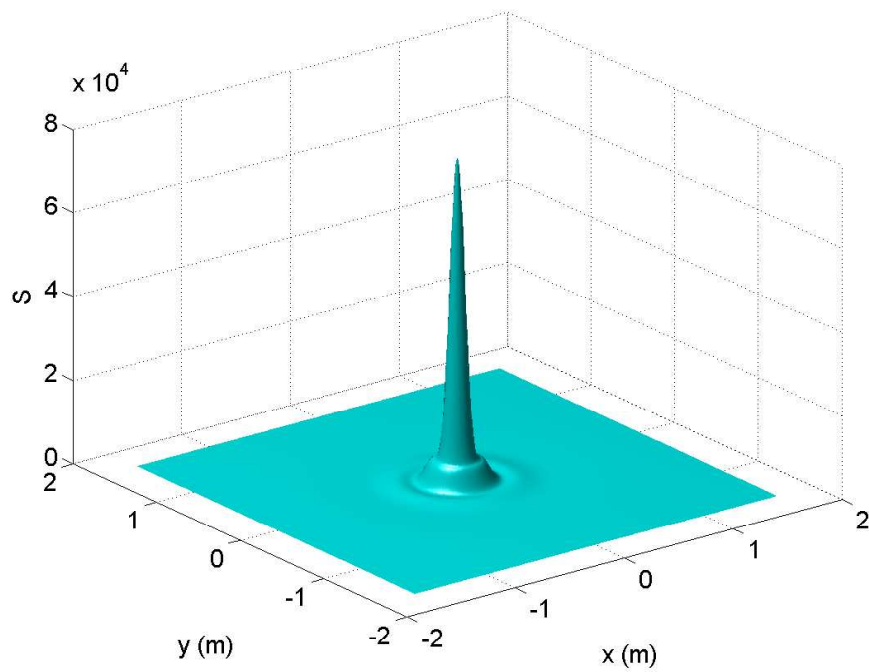


fig.12.5-16

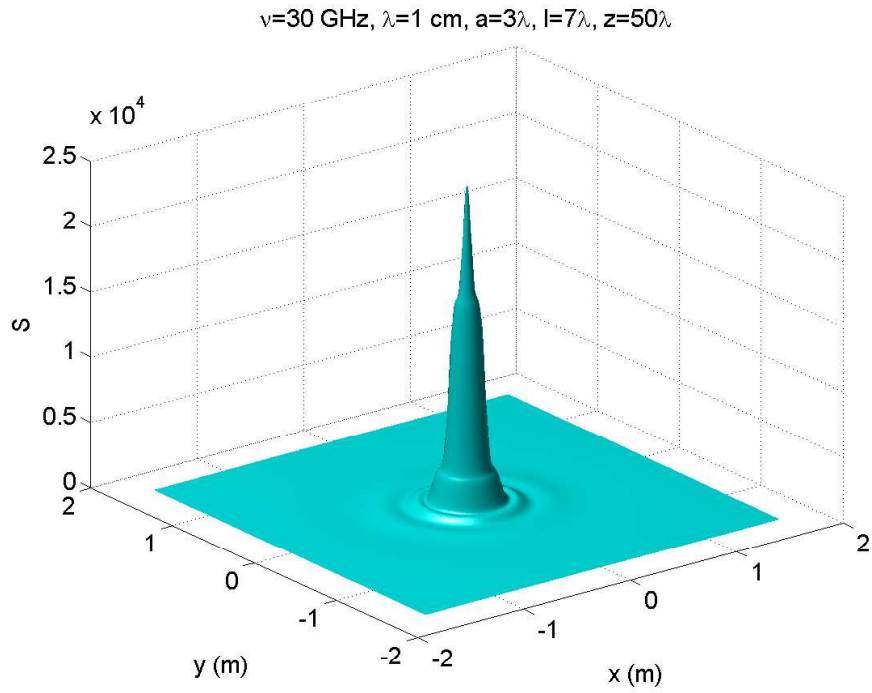


fig.12.5-17

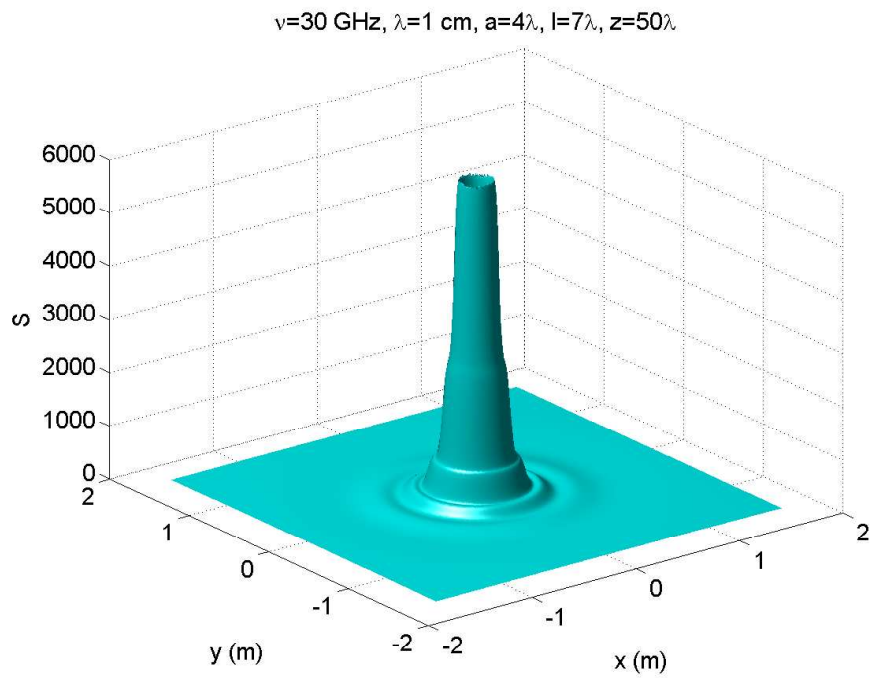


fig.12.5-18

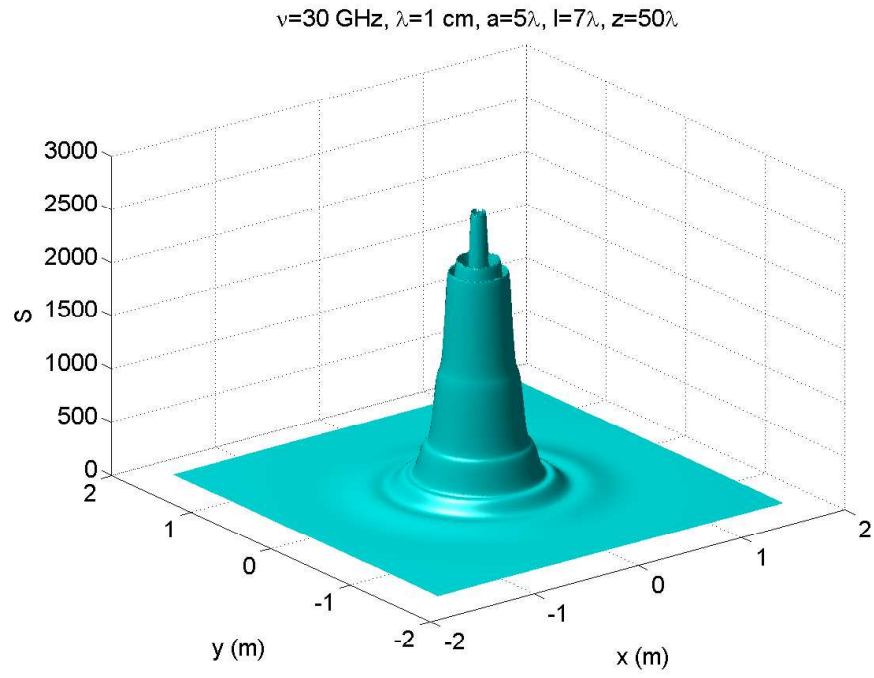


fig.12.5-19

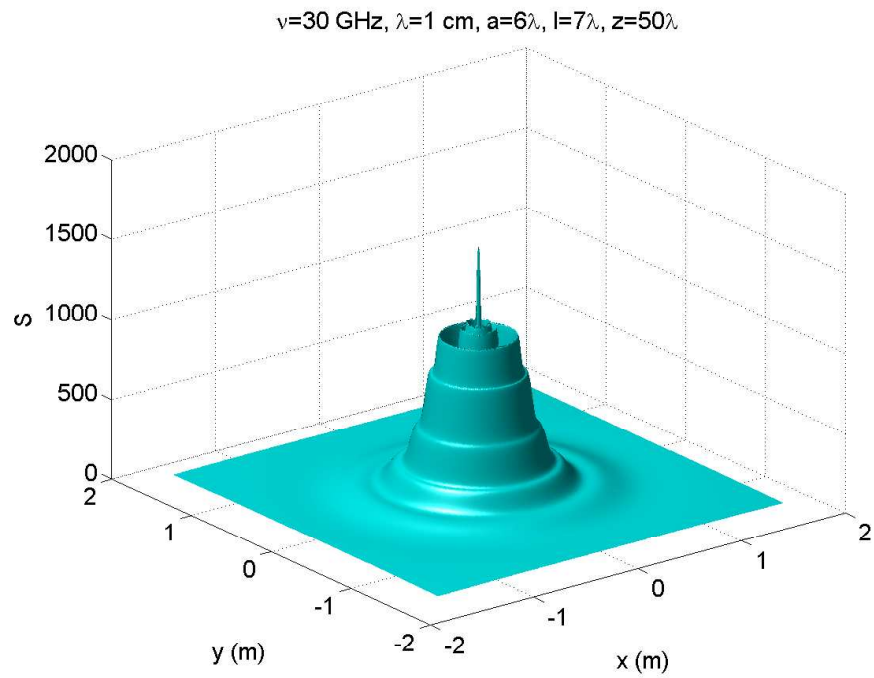


fig.12.5-20

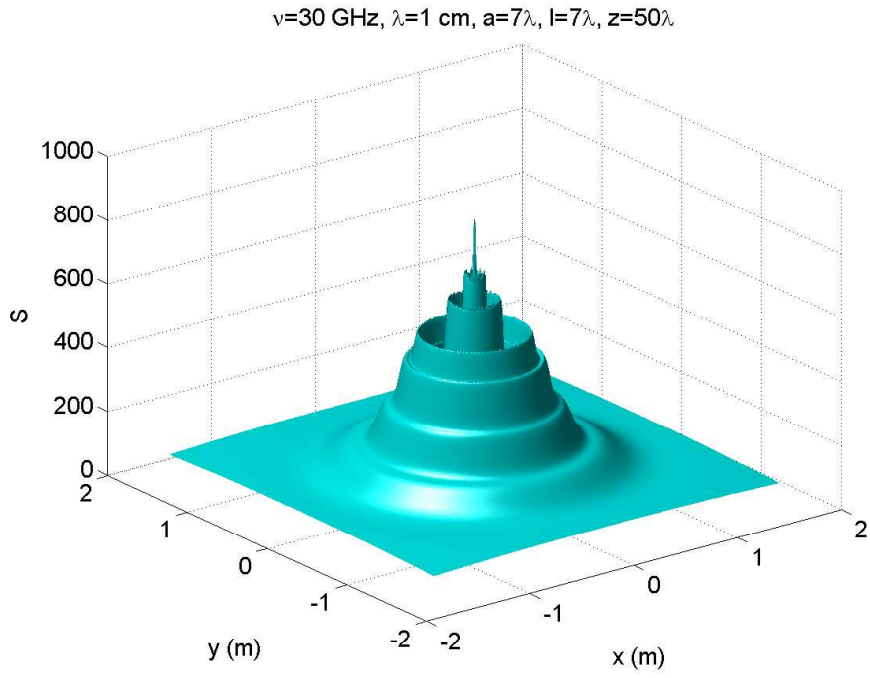


fig.12.5-21

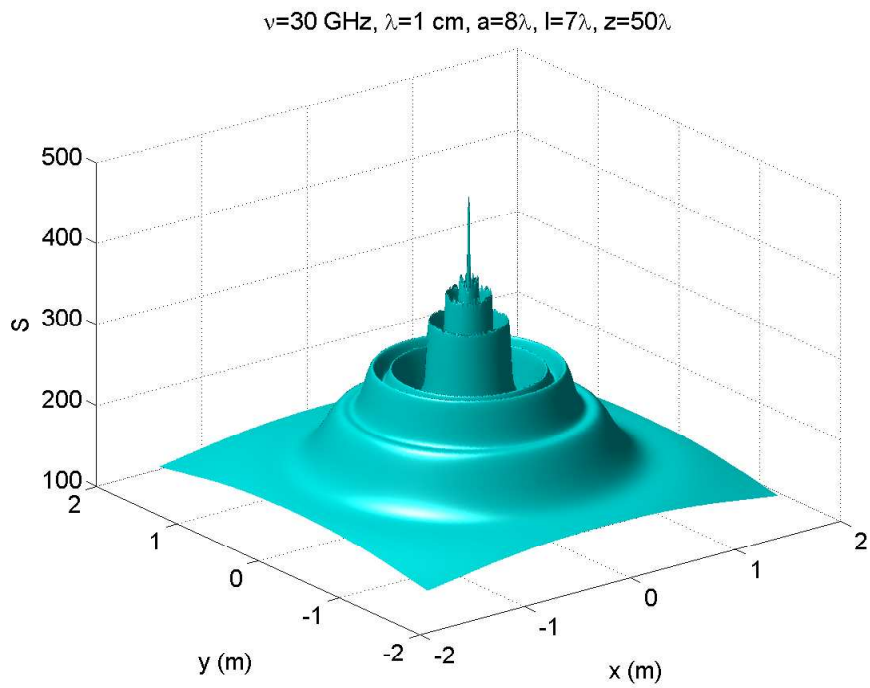


fig.12.5-22

$\nu=30\text{ GHz}$, $\lambda=1\text{ cm}$, $a=9\lambda$, $l=7\lambda$, $z=50\lambda$

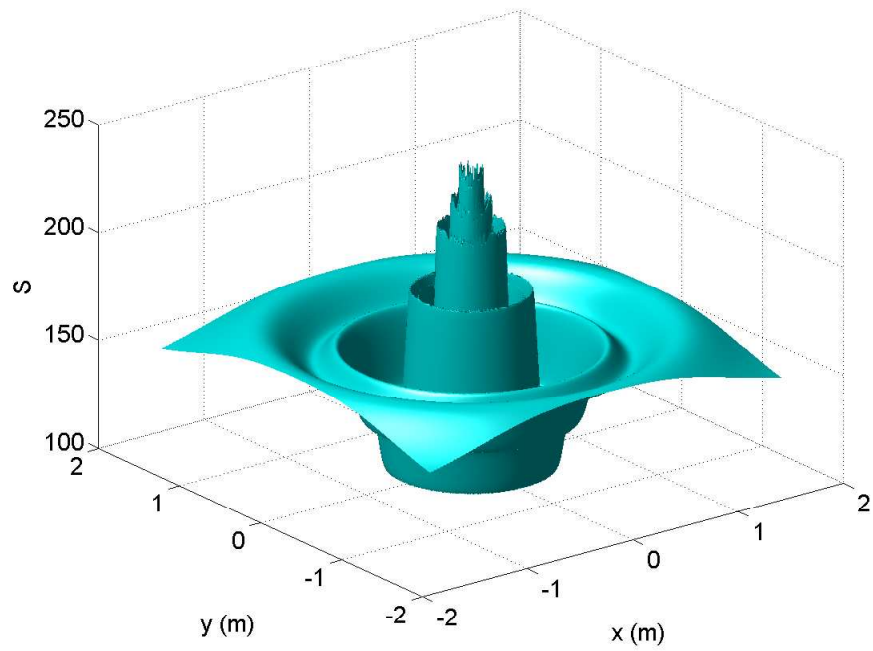


fig.12.5-23

$\nu=30\text{ GHz}$, $\lambda=1\text{ cm}$, $a=10\lambda$, $l=7\lambda$, $z=50\lambda$

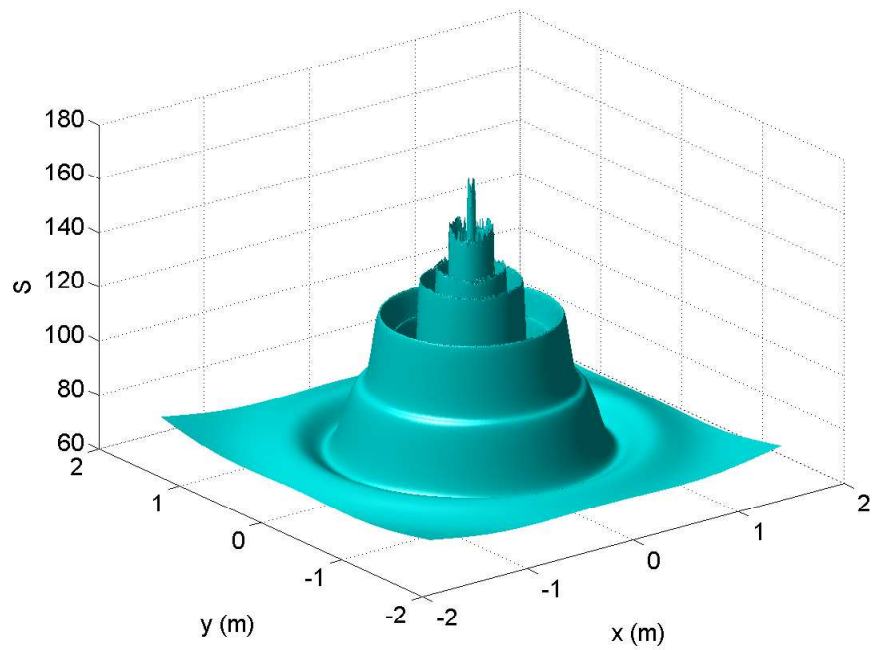


fig.12.5-24

Programmi per Diagrammi di Radiazione 3D in ambiente Matlab
Horn circolare: Diffrazione di Fraunhofer ($\nu = 10 \text{ GHz}$, $z = 50\lambda$)

$$a/\lambda = 0.5$$

```

%%%%%%%% Diffrazione da horn circolare Fraunhofer%%%%%%%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=10e9;%%%%%%%%frequenza della radiazione emessa%%%%%%%%
04 - c=3e8;
05 - lambda=c./freq;%%%%%%%%lunghezza d'onda della radiazione emessa%%%%%%%%
06 - z=50.*lambda;%%%%%%%% distanza dall'horn %%%%%%%%%
07 - a=0.5.*lambda;%%%%%%%%raggio dell'horn per freq=10GHz%%%%%%%%
08 - l=7.*lambda;%%%%%%%%lunghezza dell'horn per freq=10 GHz%%%%%%%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%%%%%%%%166.65.*lambda significa 5 metri%%%%%%%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%%%%%%%sinteta%%%%%%%%
16 - ct=z./r;%%%%%%%%costheta%%%%%%%%
17 - %%%%%%%%% Integrazione %%%%%%%%%
18 - Ris=1000; %%%%%%%%%precisione di integrazione%%%%%%%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=10 GHz, lambda=3 cm, a=0.5lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_1new')

```


$$a/\lambda = 1$$

```

%%%% Diffrazione da horn circolare Fraunhofer%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=10e9;%%frequenza della radiazione emessa%%
04 - c=3e8;
05 - lambda=c./freq;%%lunghezza d'onda della radiazione emessa%%
06 - z=50.*lambda;%%distanza dall'horn %%
07 - a=1.*lambda;%%raggio dell'horn per freq=10GHz%%
08 - l=7.*lambda;%%lunghezza dell'horn per freq=10 GHz%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%166.65.*lambda significa 5 metri%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%sintheta%%
16 - ct=z./r;%%costheta%%
17 - %% Integrazione %%
18 - Ris=1000; %%precisione di integrazione%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=10 GHz, lambda=3 cm, a=1lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_2new')

```

$$a/\lambda = 1.5$$

```

%%%% Diffrazione da horn circolare Fraunhofer%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=10e9;%%frequenza della radiazione emessa%%
04 - c=3e8;
05 - lambda=c./freq;%%lunghezza d'onda della radiazione emessa%%
06 - z=50.*lambda;%%distanza dall'horn %%
07 - a=1.5.*lambda;%%raggio dell'horn per freq=10GHz%%
08 - l=7.*lambda;%%lunghezza dell'horn per freq=10 GHz%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%166.65.*lambda significa 5 metri%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%sintheta%%
16 - ct=z./r;%%costheta%%
17 - %% Integrazione %%
18 - Ris=1000; %%precisione di integrazione%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=10 GHz, lambda=3 cm, a=1.5lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_3new')

```

$$a/\lambda = 2$$

```

%%%% Diffrazione da horn circolare Fraunhofer%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=10e9;%%frequenza della radiazione emessa%%
04 - c=3e8;
05 - lambda=c./freq;%%lunghezza d'onda della radiazione emessa%%
06 - z=50.*lambda;%%distanza dall'horn %%
07 - a=2.*lambda;%%raggio dell'horn per freq=10GHz%%
08 - l=7.*lambda;%%lunghezza dell'horn per freq=10 GHz%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%166.65.*lambda significa 5 metri%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%sintheta%%
16 - ct=z./r;%%costheta%%
17 - %% Integrazione %%
18 - Ris=1000; %%precisione di integrazione%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=10 GHz, lambda=3 cm, a=2lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_4new')

```

Programmi per Diagrammi di Radiazione 3D in ambiente Matlab
Horn circolare: Diffrazione di Fraunhofer ($\nu = 30 \text{ GHz}$, $z = 50\lambda$)

$$a/\lambda = 0.5$$

```
%%%%% Diffrazione da horn circolare Fraunhofer%%%%%%%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=30e9;%%%%%%%%frequenza della radiazione emessa%%%%%%%%
04 - c=3e8;
05 - lambda=c./freq;%%%%%%%%lunghezza d'onda della radiazione emessa%%%%%%%%
06 - z=50.*lambda;%%%%%%%% distanza dall'horn %%%%%%%%%
07 - a=0.5.*lambda;%%%%%%%%raggio dell'horn per freq=10GHz%%%%%%%%
08 - l=7.*lambda;%%%%%%%%lunghezza dell'horn per freq=10 GHz%%%%%%%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%%%%%%%%166.65.*lambda significa 1.67 metri%%%%%%%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%%%%%%%%sintheta%%%%%%%%
16 - ct=z./r;%%%%%%%%%costheta%%%%%%%%
17 - %%%%%%%%% Integrazione %%%%%%%%%
18 - Ris=1000; %%%%%%%%%%precione di integrazione%%%%%%%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=30 GHz, lambda=3 cm, a=0.5lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_5new')
```

$$a/\lambda = 1$$

```

%%%% Diffrazione da horn circolare Fraunhofer%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=30e9;%%frequenza della radiazione emessa%%
04 - c=3e8;
05 - lambda=c./freq;%%lunghezza d'onda della radiazione emessa%%
06 - z=50.*lambda;%%distanza dall'horn %%
07 - a=1.*lambda;%%raggio dell'horn per freq=10GHz%%
08 - l=7.*lambda;%%lunghezza dell'horn per freq=10 GHz%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%166.65.*lambda significa 1.67 metri%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%sintheta%%
16 - ct=z./r;%%costheta%%
17 - %% Integrazione %%
18 - Ris=1000; %%precisione di integrazione%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=30 GHz, lambda=3 cm, a=1lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_6new')

```

$$a/\lambda = 1.5$$

```

%%%% Diffrazione da horn circolare Fraunhofer%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=30e9;%%frequenza della radiazione emessa%%
04 - c=3e8;
05 - lambda=c./freq;%%lunghezza d'onda della radiazione emessa%%
06 - z=50.*lambda;%%distanza dall'horn %%
07 - a=1.5.*lambda;%%raggio dell'horn per freq=10GHz%%
08 - l=7.*lambda;%%lunghezza dell'horn per freq=10 GHz%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%166.65.*lambda significa 1.67 metri%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%sintheta%%
16 - ct=z./r;%%costheta%%
17 - %% Integrazione %%
18 - Ris=1000; %%precisione di integrazione%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=30 GHz, lambda=3 cm, a=1.5lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_7new')

```

$$a/\lambda = 2$$

```

%%%% Diffrazione da horn circolare Fraunhofer%%
01 - delete(get(0,'children'));
02 - clear all;
03 - freq=30e9;%%frequenza della radiazione emessa%%
04 - c=3e8;
05 - lambda=c./freq;%%lunghezza d'onda della radiazione emessa%%
06 - z=50.*lambda;%%distanza dall'horn %%
07 - a=2.*lambda;%%raggio dell'horn per freq=10GHz%%
08 - l=7.*lambda;%%lunghezza dell'horn per freq=10 GHz%%
09 - xp11=1.841;
10 - k=(2.*pi)./lambda;
11 - Ris1=1.*lambda;
12 - [X,Y] = meshgrid([-166.66.*lambda:Ris1:+166.66.*lambda]);
13 - %%166.65.*lambda significa 1.67 metri%%
14 - r=sqrt(X.^2+Y.^2+z.^2);
15 - s=(sqrt(X.^2+Y.^2))./r;%%sintheta%%
16 - ct=z./r;%%costheta%%
17 - %% Integrazione %%
18 - Ris=1000; %%precisione di integrazione%%
19 - wmax=1; for k1=1:Ris+1
20 - w(k1)=0+(k1-1).*((wmax-0)./Ris);
21 - end
22 - W0=0; W2=0; for j = 2:Ris+1
23 - W0=W0+w(j).*besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*s)...
24 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
25 - W2=W2+w(j).*besselj(2,w(j).*xp11).*besselj(2,k.*a.*w(j).*s)...
26 - .*exp(-i.*k.*a.^2.*w(j).^2./2./l);
27 - end
28 - Poynting=(abs(W0-W2)).^2;
29 - Poynting_z=(abs(W0-W2)).^2.*ct;
30 - Poyntingmax=max(Poynting_z);
31 - surf(X,Y,Poynting,'FaceColor','cyan','EdgeColor','None') camlight left;
32 - lighting phong
33 - set(gca,'fontsize',15);
34 - xlabel('x (m)');
35 - ylabel('y (m)');
36 - title('nu=30 GHz, lambda=3 cm, a=2lambda, l=7lambda, z=50lambda')
37 - print('-depsc2','horncircolarefraunhofer2014_8new')

```

12.6 - Diffrazione di Fresnel per l'antenna ad horn circolare

Il campo diffratto é:

$$\vec{E}(x, y, z) = \frac{1}{\lambda^2} \exp(-ikz) \int_0^a \int_0^{2\pi} \left[\hat{x}E_{ax} + \hat{y}E_{ay} - \hat{z} \left(E_{ax} \frac{x - \rho \cos \psi}{z} + E_{ay} \frac{y - \rho \sin \psi}{z} \right) \right] \exp \left[-ik \frac{(x - \rho \cos \psi)^2 + (y - \rho \sin \psi)^2}{2z} \right] \rho d\psi d\rho \quad (12.6.1)$$

Si ponga:

$$X = \int_0^a \int_0^{2\pi} E_{ax} \exp \left[-ik \frac{(x - \rho \cos \psi)^2 + (y - \rho \sin \psi)^2}{2z} \right] \rho d\psi d\rho \quad (12.6.2)$$

$$Y = \int_0^a \int_0^{2\pi} E_{ay} \exp \left[-ik \frac{(x - \rho \cos \psi)^2 + (y - \rho \sin \psi)^2}{2z} \right] \rho d\psi d\rho \quad (12.6.3)$$

Si ha:

$$\begin{aligned} E_{ax} &= E_{a\rho} \cos \psi - E_{a\psi} \sin \psi \\ E_{ay} &= E_{a\rho} \sin \psi + E_{a\psi} \cos \psi \end{aligned} \quad (12.6.4)$$

laddove:

$$\begin{aligned} E_{a\rho} &= \frac{i\omega\mu}{h^2} A_1 \frac{1}{\rho} J_1 \left(\frac{x'_{11}\rho}{a} \right) \sin \psi \exp \left(-\frac{ik\rho^2}{2l} \right) \\ E_{a\psi} &= \frac{i\omega\mu}{h} A_1 J'_1 \left(\frac{x'_{11}\rho}{a} \right) \cos \psi \exp \left(-\frac{ik\rho^2}{2l} \right) \end{aligned} \quad (12.6.5)$$

Pertanto:

$$\begin{aligned} X &= \int_0^a \int_0^{2\pi} A_1 \cos \psi \frac{i\omega\mu}{h^2} \frac{1}{\rho} J_1 \left(\frac{x'_{11}\rho}{a} \right) \sin \psi \exp \left(-\frac{ik\rho^2}{2l} \right) \cdot \\ &\cdot \exp \left[-ik \frac{(x - \rho \cos \psi)^2 + (y - \rho \sin \psi)^2}{2z} \right] \rho d\psi d\rho - \\ &- \int_0^a \int_0^{2\pi} A_1 \sin \psi \frac{i\omega\mu}{h} J'_1 \left(\frac{x'_{11}\rho}{a} \right) \cos \psi \exp \left(-\frac{ik\rho^2}{2l} \right) \cdot \\ &\cdot \exp \left[-ik \frac{(x - \rho \cos \psi)^2 + (y - \rho \sin \psi)^2}{2z} \right] \rho d\psi d\rho \end{aligned} \quad (12.6.6)$$

Si ha:

$$\begin{aligned} &\exp \left[-ik \frac{(x - \rho \cos \psi)^2 + (y - \rho \sin \psi)^2}{2z} \right] = \\ &= \exp \left[-ik \frac{r^2 \sin^2 \theta}{2z} \right] \exp \left[-ik \frac{\rho^2}{2z} \right] \exp \left[+ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] \end{aligned} \quad (12.6.7)$$

Quindi:

$$\begin{aligned}
 X &= A_1 \frac{i\omega\mu}{h^2} \exp \left[-ik \frac{r^2 \sin^2 \theta}{2z} \right] \int_0^a \int_0^{2\pi} \cos \psi \sin \psi J_1 \left(\frac{x'_{11}\rho}{a} \right) \exp \left(-\frac{ik\rho^2}{2l} \right) \cdot \\
 &\cdot \exp \left[-ik \frac{\rho^2}{2z} \right] \exp \left[+ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] d\psi d\rho - \\
 &- A_1 \frac{i\omega\mu}{h} \exp \left[-ik \frac{r^2 \sin^2 \theta}{2z} \right] \int_0^a \int_0^{2\pi} \cos \psi \sin \psi J'_1 \left(\frac{x'_{11}\rho}{a} \right) \exp \left(-\frac{ik\rho^2}{2l} \right) \cdot \\
 &\cdot \exp \left[-ik \frac{\rho^2}{2z} \right] \exp \left[+ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] \rho d\psi d\rho
 \end{aligned} \tag{12.6.8}$$

$$\begin{aligned}
 X &= A_1 i\omega\mu \exp \left[-ik \frac{r^2 \sin^2 \theta}{2z} \right] \int_0^a \int_0^{2\pi} \cos \psi \sin \psi \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] \cdot \\
 &\cdot \exp \left[+ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] \left[\frac{1}{h^2} J_1 \left(\frac{x'_{11}\rho}{a} \right) - \frac{\rho}{h} J'_1 \left(\frac{x'_{11}\rho}{a} \right) \right] d\psi d\rho = \\
 &= A_1 i\omega\mu \exp \left[-ik \frac{r^2 \sin^2 \theta}{2z} \right] \int_0^a \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] \left[\frac{1}{h^2} J_1 \left(\frac{x'_{11}\rho}{a} \right) - \frac{\rho}{h} J'_1 \left(\frac{x'_{11}\rho}{a} \right) \right] \cdot \\
 &\cdot \left\{ \int_0^{2\pi} \cos \psi \sin \psi \exp \left[+ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] d\psi \right\} d\rho
 \end{aligned} \tag{12.6.9}$$

Applicando lo sviluppo di Jacobi:

$$\begin{aligned}
 \exp \left[ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] &= J_0 \left(\frac{kr}{z} \rho \sin \theta \right) + 2 \sum_{n=1}^{+\infty} i^n J_n \left(\frac{kr}{z} \rho \sin \theta \right) \cos(n\phi) \cos(n\psi) + \\
 &+ 2 \sum_{n=1}^{+\infty} i^n J_n \left(\frac{kr}{z} \rho \sin \theta \right) \sin(n\phi) \sin(n\psi)
 \end{aligned} \tag{12.6.10}$$

Quindi l'integrale in $d\psi$ diventa:

$$\begin{aligned}
 \int_0^{2\pi} \cos \psi \sin \psi \exp \left[ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] d\psi &= J_0 \left(\frac{kr}{z} \rho \sin \theta \right) \int_0^{2\pi} \cos \psi \sin \psi d\psi + \\
 + 2 \sum_{n=1}^{+\infty} i^n \cos n\phi J_n \left(\frac{kr}{z} \rho \sin \theta \right) \int_0^{2\pi} \cos \psi \sin \psi \cos n\psi d\psi &+ \\
 + 2 \sum_{n=1}^{+\infty} i^n \sin n\phi J_n \left(\frac{kr}{z} \rho \sin \theta \right) \int_0^{2\pi} \cos \psi \sin \psi \sin n\psi d\psi &
 \end{aligned} \tag{12.6.11}$$

$$\begin{aligned}
 X &= A_1 i \omega \mu \exp \left[-ik \frac{r^2 \sin^2 \theta}{2z} \right] \cdot \\
 &\cdot \int_0^a \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] \left[\frac{1}{h^2} J_1 \left(\frac{x'_{11}\rho}{a} \right) - \frac{\rho}{h} J'_1 \left(\frac{x'_{11}\rho}{a} \right) \right] \left[-\pi \sin 2\phi J_2 \left(\frac{kr}{z} \rho \sin \theta \right) \right] d\rho = \\
 &= -A_1 \pi i \omega \mu \sin 2\phi \exp \left[-ik \frac{r^2 \sin^2 \theta}{2z} \right] \cdot \\
 &\cdot \int_0^a \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] \left[\frac{1}{h^2} J_1 \left(\frac{x'_{11}\rho}{a} \right) - \frac{\rho}{h} J'_1 \left(\frac{x'_{11}\rho}{a} \right) \right] J_2 \left(\frac{kr}{z} \rho \sin \theta \right) d\rho
 \end{aligned} \tag{12.6.12}$$

Consideriamo la seguente formula ricorrente:

$$z J'_1(z) - J_1(z) = -z J_2(z) \tag{12.6.13}$$

Poniamo $z = \frac{x'_{11}\rho}{a}$ e moltiplichiamo per $\exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] J_2 \left(\frac{kr}{z} \rho \sin \theta \right)$ entrambi i membri della (12.6.13). Si ha:

$$\begin{aligned}
 &\frac{x'_{11}\rho}{a} \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] J_2 \left(\frac{kr}{z} \rho \sin \theta \right) J'_1 \left(\frac{x'_{11}\rho}{a} \right) - \\
 &- \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] J_2 \left(\frac{kr}{z} \rho \sin \theta \right) J_1 \left(\frac{x'_{11}\rho}{a} \right) = \\
 &= -\frac{x'_{11}\rho}{a} \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] J_2 \left(\frac{kr}{z} \rho \sin \theta \right) J_2 \left(\frac{x'_{11}\rho}{a} \right)
 \end{aligned} \tag{12.6.14}$$

Quindi:

$$\begin{aligned}
 X &= A_1 \frac{\pi i \omega \mu}{h^2} \sin 2\phi \exp \left(-ik \frac{r^2 \sin^2 \theta}{2z} \right) \cdot \\
 &\cdot \int_0^a \frac{x'_{11}}{a} \exp \left[-\frac{ik\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] J_2 \left(\frac{x'_{11}\rho}{a} \right) J_2 \left(\frac{kr}{z} \rho \sin \theta \right) \rho d\rho
 \end{aligned} \tag{12.6.15}$$

Posto $\frac{\rho}{a} = w$, si ha:

$$\begin{aligned}
 X &= A_1 \frac{\pi i \omega \mu}{h^2} \sin 2\phi \exp \left(-ik \frac{r^2 \sin^2 \theta}{2z} \right) \cdot \\
 &\cdot \int_0^1 \frac{x'_{11}}{a} \exp \left[-\frac{ika^2 w^2}{2} \left(\frac{1}{l} + \frac{1}{z} \right) \right] J_2 (x'_{11} w) J_2 \left(\frac{kr}{z} w a \sin \theta \right) w a^2 dw
 \end{aligned}$$

(12.6.16)

Calcoliamo, ora, la componente lungo l'asse y.

$$\begin{aligned}
 Y &= \int_0^a \int_0^{2\pi} iA_1 \frac{\omega\mu}{h^2} \sin\psi \frac{1}{\rho} J_1\left(\frac{x'_{11}\rho}{a}\right) \sin\psi \exp\left(-i\frac{k\rho^2}{2l}\right) \cdot \\
 &\cdot \exp\left[-ik\frac{(x-\rho\cos\psi)^2+(y-\rho\sin\psi)^2}{2z}\right] \rho d\psi d\rho + \\
 &+ \int_0^a \int_0^{2\pi} iA_1 \frac{\omega\mu}{h} \cos\psi J'_1\left(\frac{x'_{11}\rho}{a}\right) \cos\psi \exp\left(-i\frac{k\rho^2}{2l}\right) \cdot \\
 &\cdot \exp\left[-ik\frac{(x-\rho\cos\psi)^2+(y-\rho\sin\psi)^2}{2z}\right] \rho d\psi d\rho = \\
 &= \int_0^a \int_0^{2\pi} iA_1 \frac{\omega\mu}{h^2} \sin^2\psi \frac{1}{\rho} J_1\left(\frac{x'_{11}\rho}{a}\right) \exp\left(-i\frac{k\rho^2}{2l}\right) \cdot \\
 &\cdot \exp\left[-ik\frac{(x-\rho\cos\psi)^2+(y-\rho\sin\psi)^2}{2z}\right] \rho d\psi d\rho + \\
 &+ \int_0^a \int_0^{2\pi} iA_1 \frac{\omega\mu}{h} \cos^2\psi J'_1\left(\frac{x'_{11}\rho}{a}\right) \exp\left(-i\frac{k\rho^2}{2l}\right) \cdot \\
 &\cdot \exp\left[-ik\frac{(x-\rho\cos\psi)^2+(y-\rho\sin\psi)^2}{2z}\right] \rho d\psi d\rho = \\
 &= iA_1 \frac{\omega\mu}{h} \int_0^a \int_0^{2\pi} \exp\left(-i\frac{k\rho^2}{2l}\right) \exp\left[-ik\frac{(x-\rho\cos\psi)^2+(y-\rho\sin\psi)^2}{2z}\right] \cdot \\
 &\cdot \left[\sin^2\psi \frac{1}{h} J_1\left(\frac{x'_{11}\rho}{a}\right) + \rho \cos\psi J'_1\left(\frac{x'_{11}\rho}{a}\right)\right] d\psi d\rho
 \end{aligned} \tag{12.6.17}$$

Si ha:

$$\begin{aligned}
 \exp\left[-ik\frac{(x-\rho\cos\psi)^2+(y-\rho\sin\psi)^2}{2z}\right] &= \exp\left(-ik\frac{r^2\sin^2\theta}{2z}\right) \cdot \\
 \cdot \exp\left(-ik\frac{\rho^2}{2z}\right) \exp\left[ik\frac{\rho r \sin\theta \cos(\phi-\psi)}{z}\right]
 \end{aligned} \tag{12.6.18}$$

Pertanto l'equazione (12.6.17) si può scrivere:

$$\begin{aligned}
 & iA_1 \frac{\omega\mu}{h} \int_0^a \int_0^{2\pi} \exp\left(-i\frac{k\rho^2}{2l}\right) \exp\left(-ik\frac{r^2 \sin^2 \theta}{2z}\right) \cdot \\
 & \cdot \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] \left[\sin^2 \psi \frac{1}{h} J_1\left(\frac{x'_{11}\rho}{a}\right) + \cos^2 \psi J'_1\left(\frac{x'_{11}\rho}{a}\right)\right] d\psi d\rho = \\
 & = iA_1 \frac{\omega\mu}{h} \exp\left(-ik\frac{r^2 \sin^2 \theta}{2z}\right) \left\{ \int_0^a \int_0^{2\pi} \exp\left[-i\frac{k\rho^2}{2}\left(\frac{1}{l} + \frac{1}{z}\right)\right] \cdot \right. \\
 & \cdot \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] \sin^2 \psi \frac{1}{h} J_1\left(\frac{x'_{11}\rho}{a}\right) d\rho d\psi + \\
 & + \int_0^a \int_0^{2\pi} \exp\left[-i\frac{k\rho^2}{2}\left(\frac{1}{l} + \frac{1}{z}\right)\right] \cdot \\
 & \cdot \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] \cos^2 \psi J'_1\left(\frac{x'_{11}\rho}{a}\right) \rho d\rho d\psi \left. \right\} = \\
 & = iA_1 \frac{\omega\mu}{h} \exp\left(-ik\frac{r^2 \sin^2 \theta}{2z}\right) \left\{ \frac{1}{h} \int_0^a \exp\left[-i\frac{k\rho^2}{2}\left(\frac{1}{l} + \frac{1}{z}\right)\right] J_1\left(\frac{x'_{11}\rho}{a}\right) \cdot \right. \\
 & \cdot \int_0^{2\pi} \sin^2 \psi \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] d\rho d\psi + \\
 & + \int_0^a \exp\left[-i\frac{k\rho^2}{2}\left(\frac{1}{l} + \frac{1}{z}\right)\right] J'_1\left(\frac{x'_{11}\rho}{a}\right) \cdot \\
 & \cdot \int_0^{2\pi} \cos^2 \psi \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] d\rho d\psi \left. \right\}
 \end{aligned} \tag{12.6.19}$$

Calcoliamo a parte gli integrali:

$$\int_0^{2\pi} \sin^2 \psi \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] d\psi \quad \text{e} \quad \int_0^{2\pi} \cos^2 \psi \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] d\psi \tag{12.6.20}$$

Applicando lo sviluppo di Jacobi dato dall'equazione (12.6.10) si ha:

$$\begin{aligned}
 & \int_0^{2\pi} \sin^2 \psi \exp\left[ik\frac{\rho r \sin \theta \cos(\phi - \psi)}{z}\right] d\psi = \int_0^{2\pi} \sin^2 \psi J_0\left(\frac{kr}{z}\rho \sin \theta\right) d\psi + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n\left(\frac{kr}{z}\rho \sin \theta\right) \cos n\phi \int_0^{2\pi} \sin^2 \psi \cos n\psi d\psi + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n\left(\frac{kr}{z}\rho \sin \theta\right) \sin n\phi \int_0^{2\pi} \sin^2 \psi \sin n\psi d\psi
 \end{aligned} \tag{12.6.21}$$

che si può ancora scrivere:

$$\begin{aligned}
 & J_0 \left(\frac{kr}{z} \rho \sin \theta \right) \int_0^{2\pi} \sin^2 \psi d\psi + 2 \sum_{n=1}^{+\infty} i^n J_n \left(\frac{kr}{z} \rho \sin \theta \right) \cos n\phi \int_0^{2\pi} \sin^2 \psi \cos n\psi d\psi + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n \left(\frac{kr}{z} \rho \sin \theta \right) \sin n\phi \int_0^{2\pi} \sin^2 \psi \sin n\psi d\psi = \\
 & = \pi J_0 \left(\frac{kr}{z} \rho \sin \theta \right) + \pi \cos 2\phi J_2 \left(\frac{kr}{z} \rho \sin \theta \right)
 \end{aligned} \tag{12.6.22}$$

In definitiva:

$$\int_0^{2\pi} \sin^2 \psi \exp \left[ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] d\psi = \pi J_0 \left(\frac{kr}{z} \rho \sin \theta \right) +$$

$$+ \pi \cos 2\phi J_2 \left(\frac{kr}{z} \rho \sin \theta \right)$$

(12.6.23)

Analogamente:

$$\begin{aligned}
 & \int_0^{2\pi} \cos^2 \psi \exp \left[ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] d\psi = J_0 \left(\frac{kr}{z} \rho \sin \theta \right) \int_0^{2\pi} \cos^2 \psi d\psi + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n \left(\frac{kr}{z} \rho \sin \theta \right) \cos n\phi \int_0^{2\pi} \cos^2 \psi \cos n\psi d\psi + \\
 & + 2 \sum_{n=1}^{+\infty} i^n J_n \left(\frac{kr}{z} \rho \sin \theta \right) \sin n\phi \int_0^{2\pi} \cos^2 \psi \sin n\psi d\psi = \\
 & = \pi J_0 \left(\frac{kr}{z} \rho \sin \theta \right) - \pi \cos 2\phi J_2 \left(\frac{kr}{z} \rho \sin \theta \right)
 \end{aligned} \tag{12.6.24}$$

In definitiva:

$$\int_0^{2\pi} \cos^2 \psi \exp \left[ik \frac{\rho r \sin \theta \cos(\phi - \psi)}{z} \right] d\psi = \pi J_0 \left(\frac{kr}{z} \rho \sin \theta \right) -$$

$$- \pi \cos 2\phi J_2 \left(\frac{kr}{z} \rho \sin \theta \right)$$

(12.6.25)

Quindi la (12.6.19) si scrive:

$$\begin{aligned}
 & iA_1 \frac{\omega\mu}{h} \exp\left(-ik \frac{r^2 \sin^2 \theta}{2z}\right) \left\{ \frac{1}{h} \int_0^a \exp\left[-i \frac{k\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] J_1\left(\frac{x'_{11}\rho}{a}\right) \cdot \right. \\
 & \cdot \left[\pi J_0\left(\frac{kr}{z} \rho \sin \theta\right) + \pi \cos 2\phi J_2\left(\frac{kr}{z} \rho \sin \theta\right) \right] d\rho + \\
 & + \int_0^a \exp\left[-i \frac{k\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] J'_1\left(\frac{x'_{11}\rho}{a}\right) \rho \cdot \\
 & \cdot \left[\pi J_0\left(\frac{kr}{z} \rho \sin \theta\right) - \pi \cos 2\phi J_2\left(\frac{kr}{z} \rho \sin \theta\right) \right] d\rho \left. \right\} = \\
 & = iA_1 \frac{\omega\mu}{h} \exp\left(-ik \frac{r^2 \sin^2 \theta}{2z}\right) \pi \left\{ \int_0^a \exp\left[-i \frac{k\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] J_0\left(\frac{kr}{z} \rho \sin \theta\right) \cdot \right. \\
 & \cdot \left[\frac{1}{h} J_1\left(\frac{x'_{11}\rho}{a}\right) + \rho J'_1\left(\frac{x'_{11}\rho}{a}\right) \right] d\rho + \\
 & + \int_0^a \exp\left[-i \frac{k\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] \cos 2\phi J_2\left(\frac{kr}{z} \rho \sin \theta\right) \cdot \\
 & \cdot \left[\frac{1}{h} J_1\left(\frac{x'_{11}\rho}{a}\right) - \rho J'_1\left(\frac{x'_{11}\rho}{a}\right) \right] d\rho \left. \right\} =
 \end{aligned} \tag{12.6.26}$$

$$\begin{aligned}
 & = iA_1 \frac{\omega\mu}{h} \exp\left(-ik \frac{r^2 \sin^2 \theta}{2z}\right) \pi \left\{ \int_0^a \exp\left[-i \frac{k\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] \cdot \right. \\
 & \cdot \rho J_0\left(\frac{kr \sin \theta}{z} \rho\right) J_0\left(\frac{x'_{11}\rho}{a}\right) d\rho + \\
 & + \cos 2\phi \int_0^a \exp\left[-i \frac{k\rho^2}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] J_2\left(\frac{kr}{z} \rho \sin \theta\right) J_2\left(\frac{x'_{11}\rho}{a}\right) \rho d\rho \left. \right\}
 \end{aligned}$$

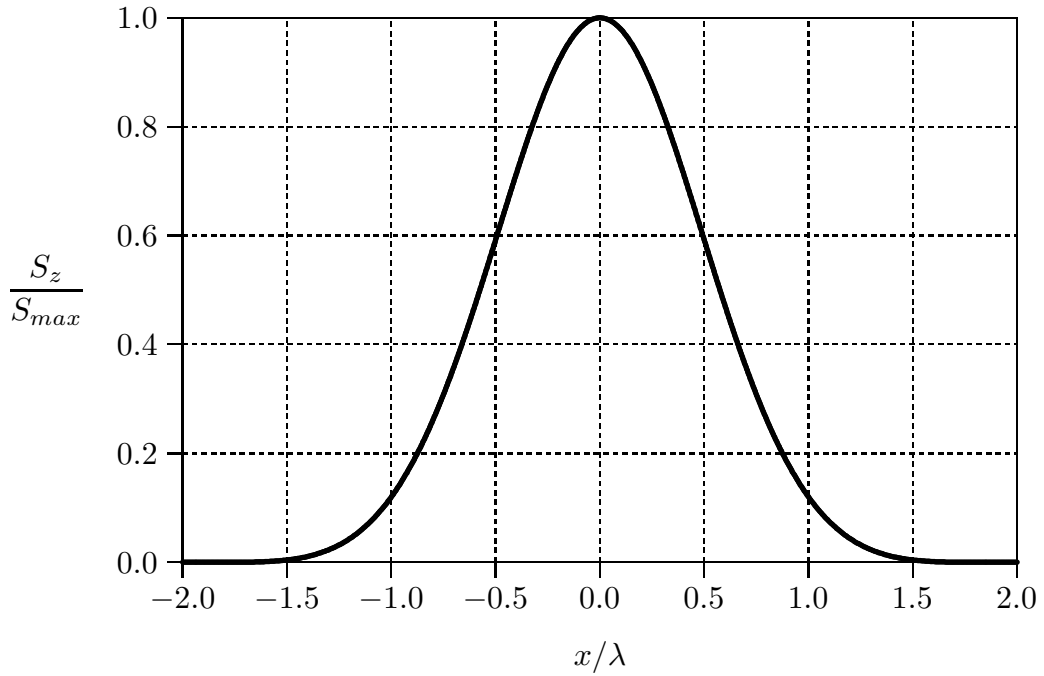
Effettuando la solita trasformazione si ha:

$$\begin{aligned}
 Y & = iA_1 \frac{\omega\mu}{h^2} \exp\left(-ik \frac{r^2 \sin^2 \theta}{2z}\right) \pi \left\{ \int_0^1 \exp\left[-i \frac{ka^2 w r}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] \cdot \right. \\
 & \cdot \rho J_0\left(\frac{kr \sin \theta}{z} a w\right) J_0(x'_{11} w) d\rho + \\
 & + \cos 2\phi \int_0^1 \exp\left[-i \frac{ka^2 w^2}{2} \left(\frac{1}{l} + \frac{1}{z}\right)\right] J_2\left(\frac{kr}{z} a w \sin \theta\right) J_2(x'_{11} w) \rho d\rho \left. \right\}
 \end{aligned}$$

(12.6.27)

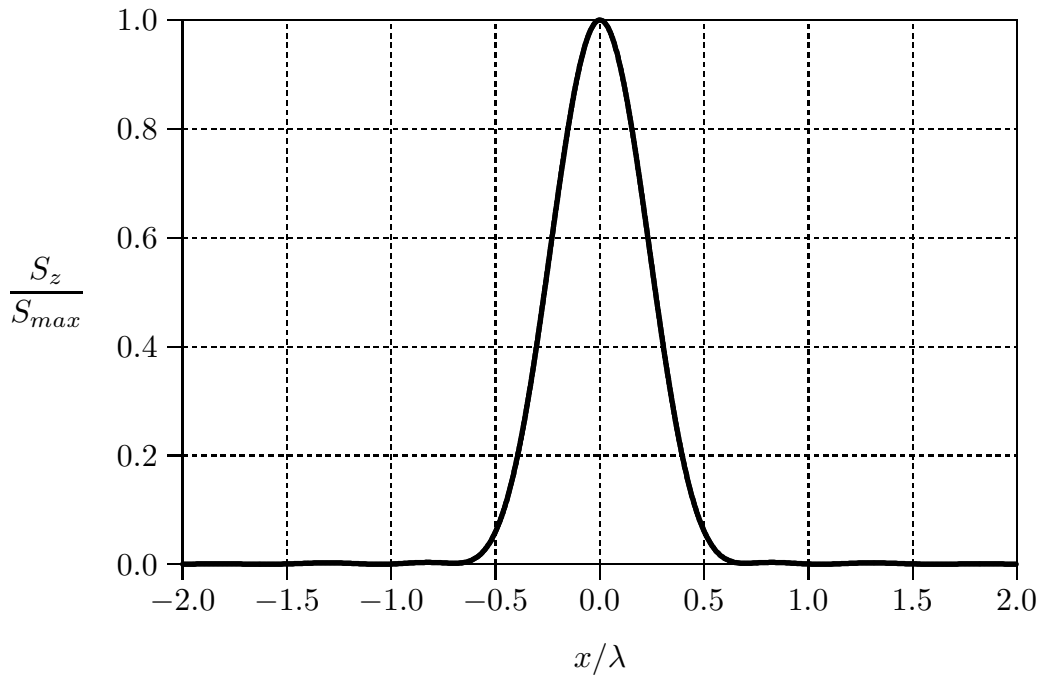
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 0.5\lambda$, $l = 7\lambda$, $z = 1\lambda$



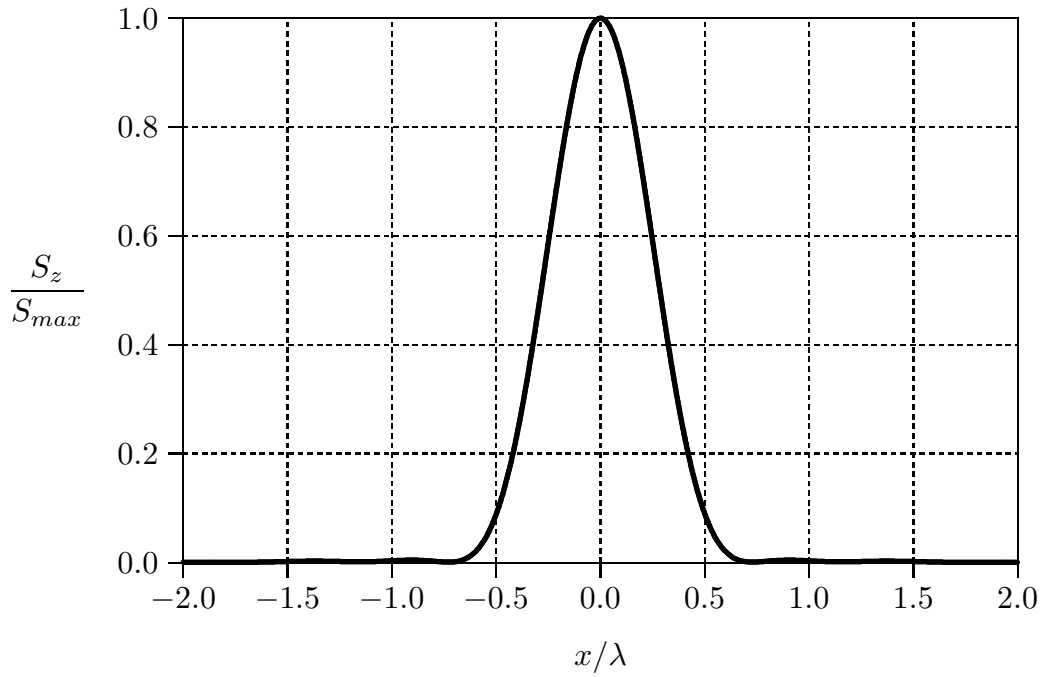
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 1\lambda$, $l = 7\lambda$, $z = 1\lambda$



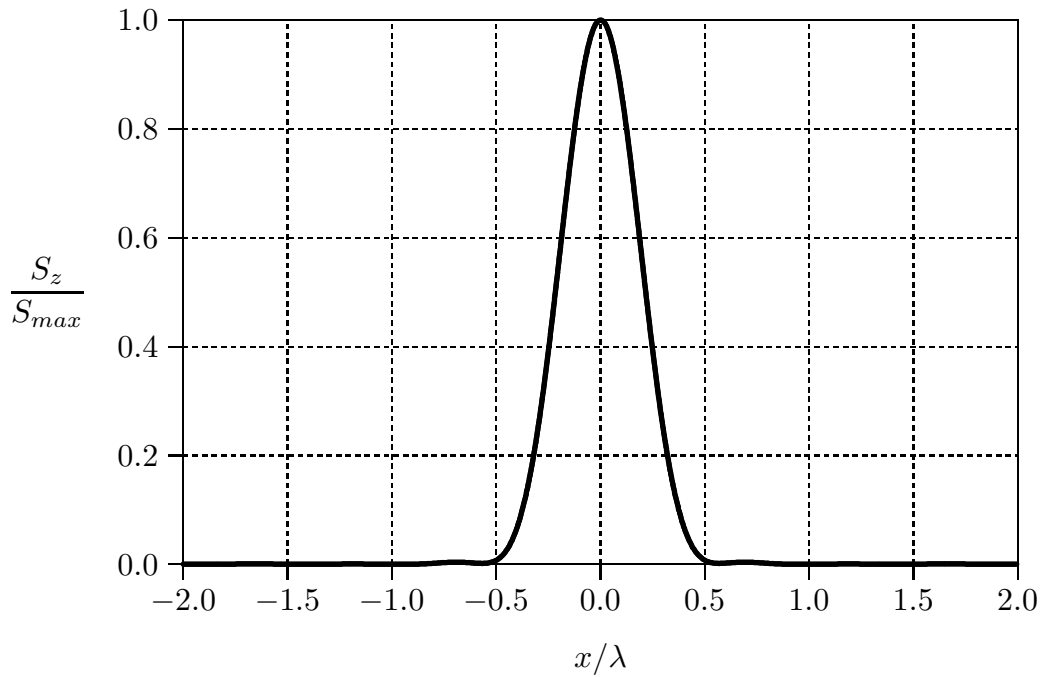
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 1.5\lambda$, $l = 7\lambda$, $z = 1\lambda$



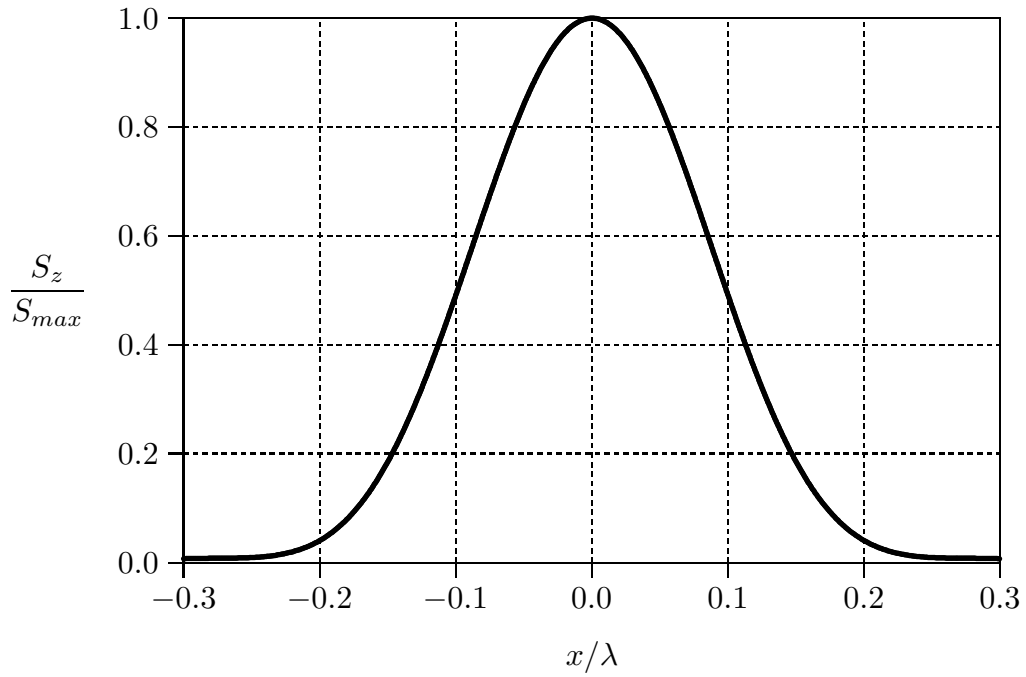
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 2\lambda$, $l = 7\lambda$, $z = 1\lambda$



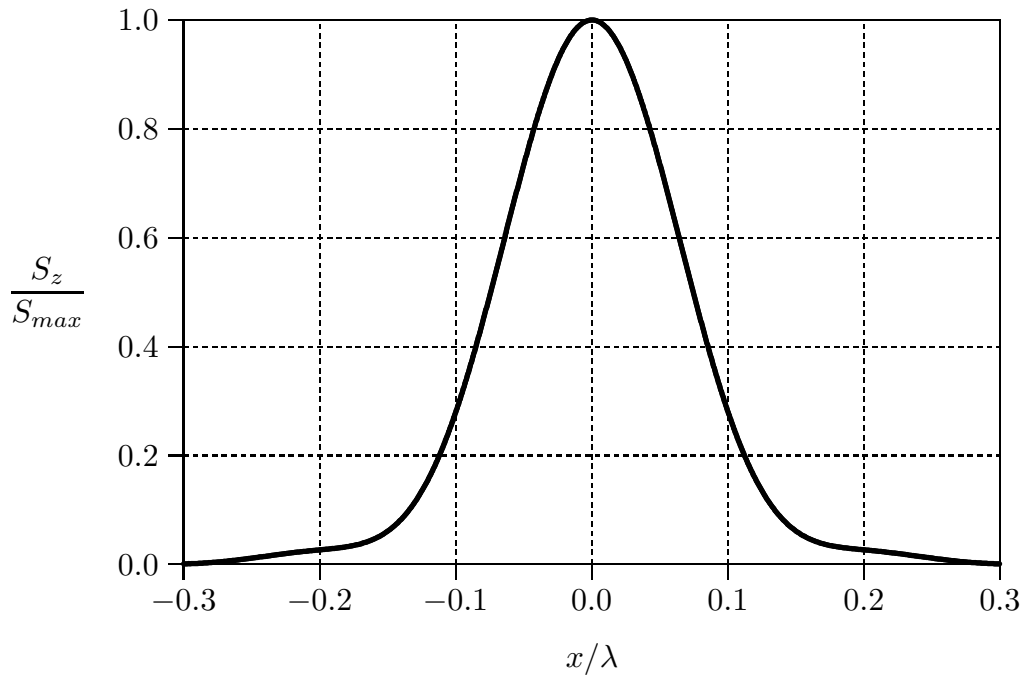
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 3\lambda$, $l = 7\lambda$, $z = 1\lambda$



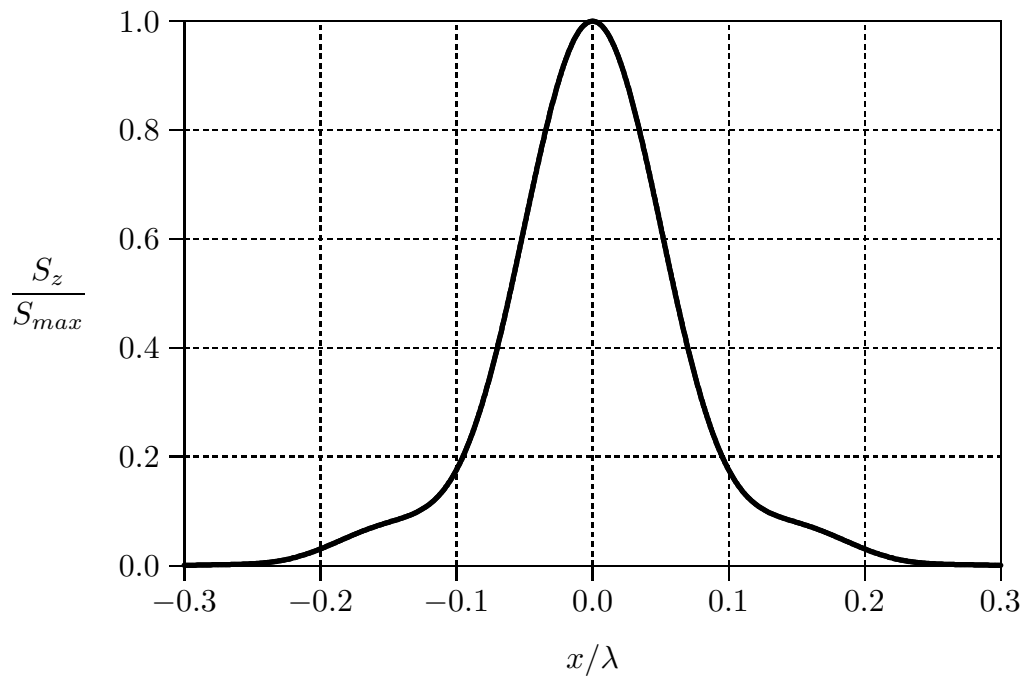
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 4\lambda$, $l = 7\lambda$, $z = 1\lambda$



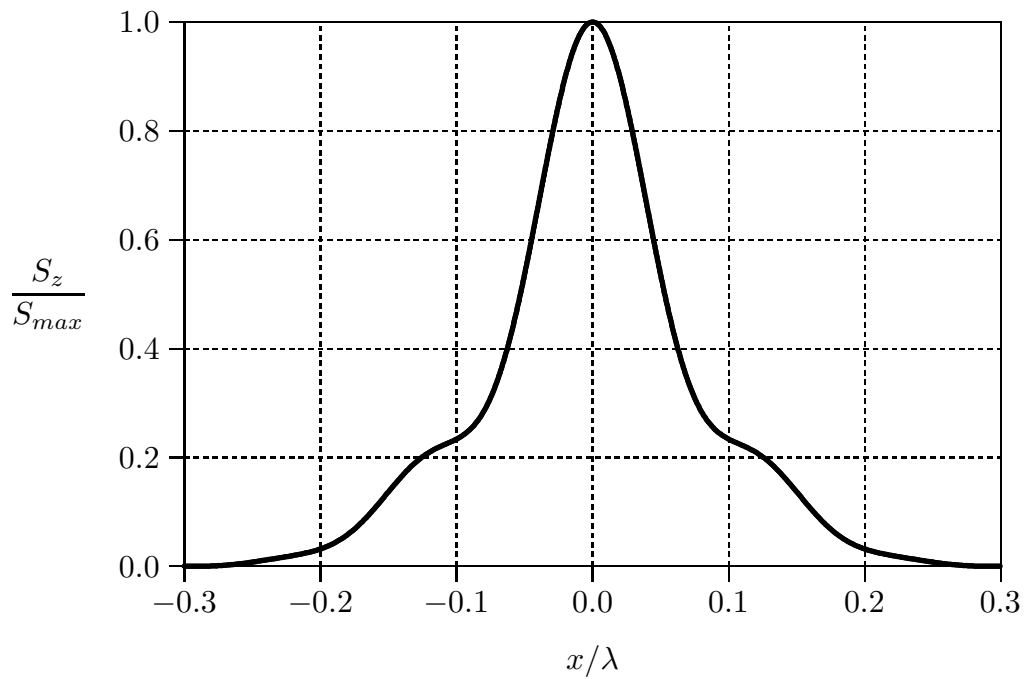
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 5\lambda$, $l = 7\lambda$, $z = 1\lambda$



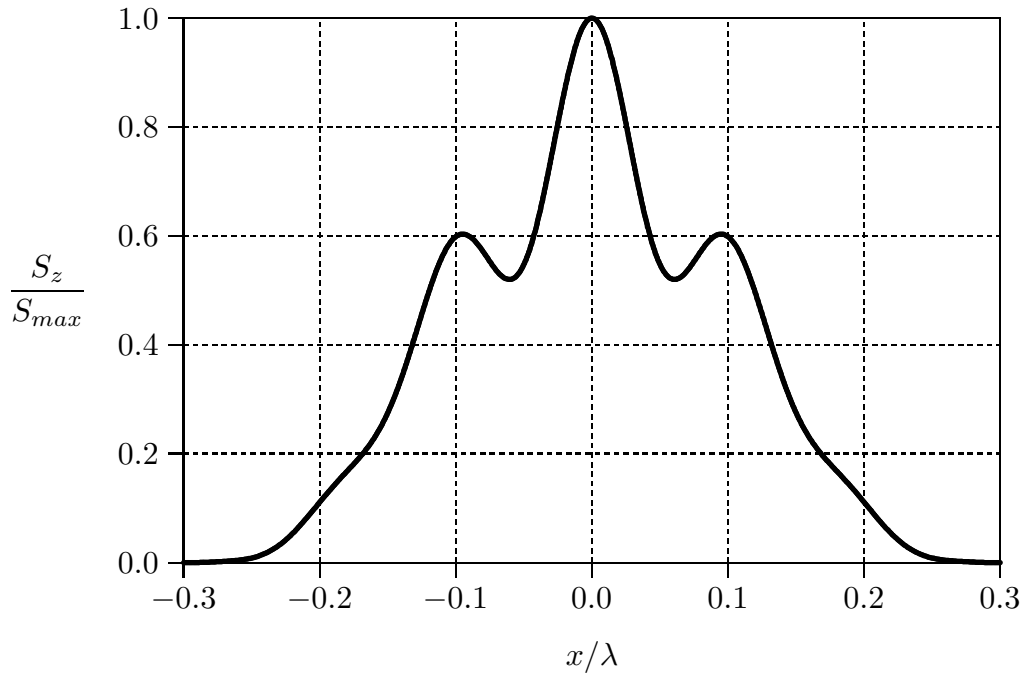
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 6\lambda$, $l = 7\lambda$, $z = 1\lambda$



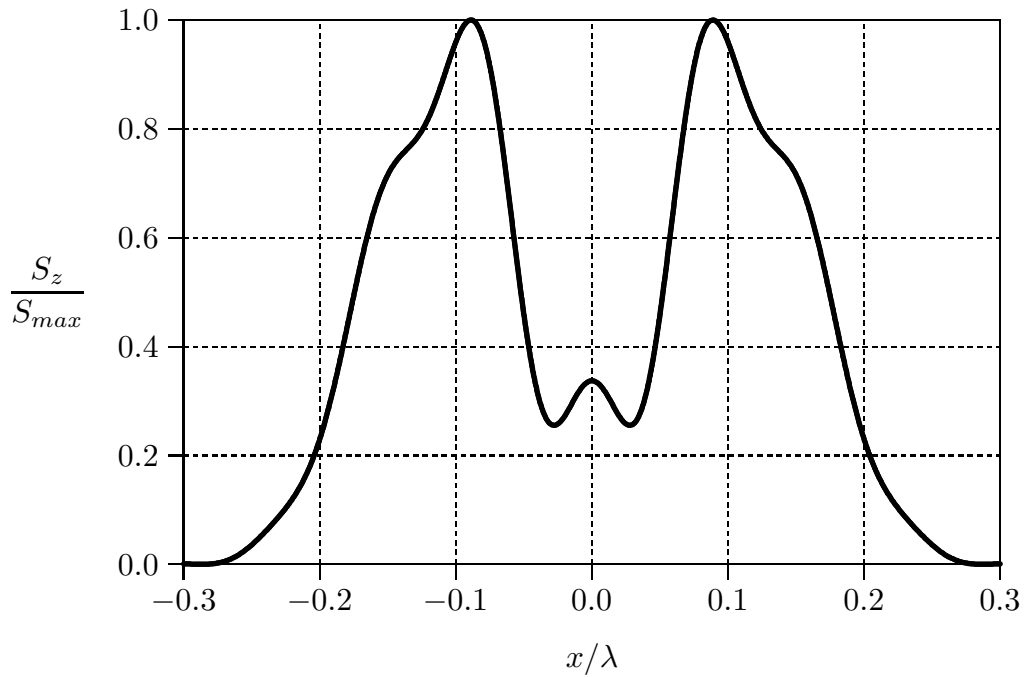
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 7\lambda$, $l = 7\lambda$, $z = 1\lambda$



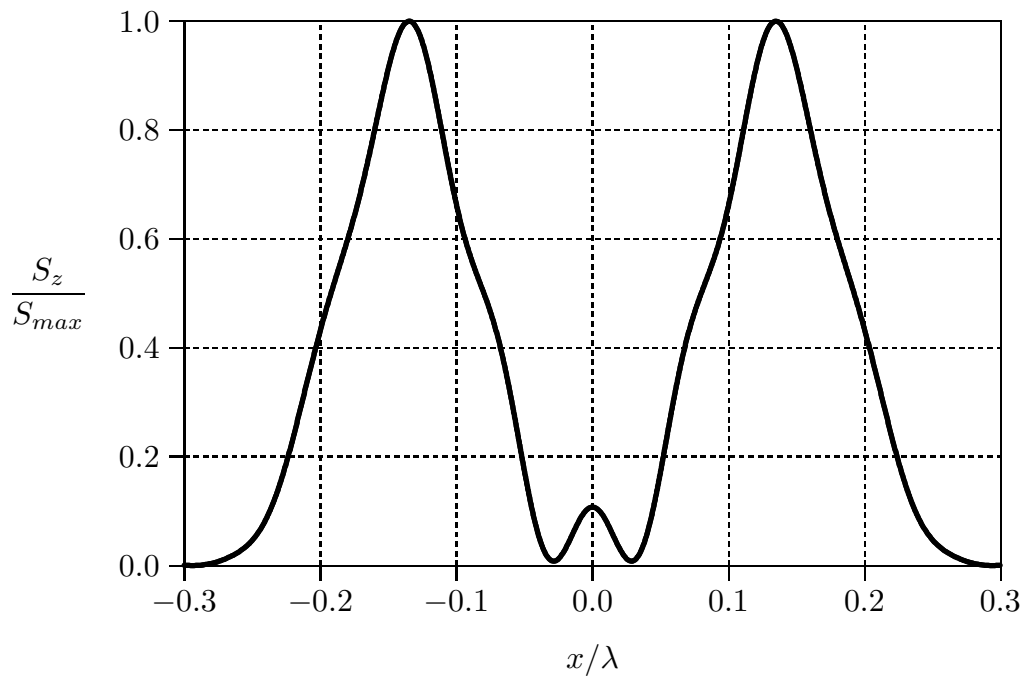
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 8\lambda$, $l = 7\lambda$, $z = 1\lambda$



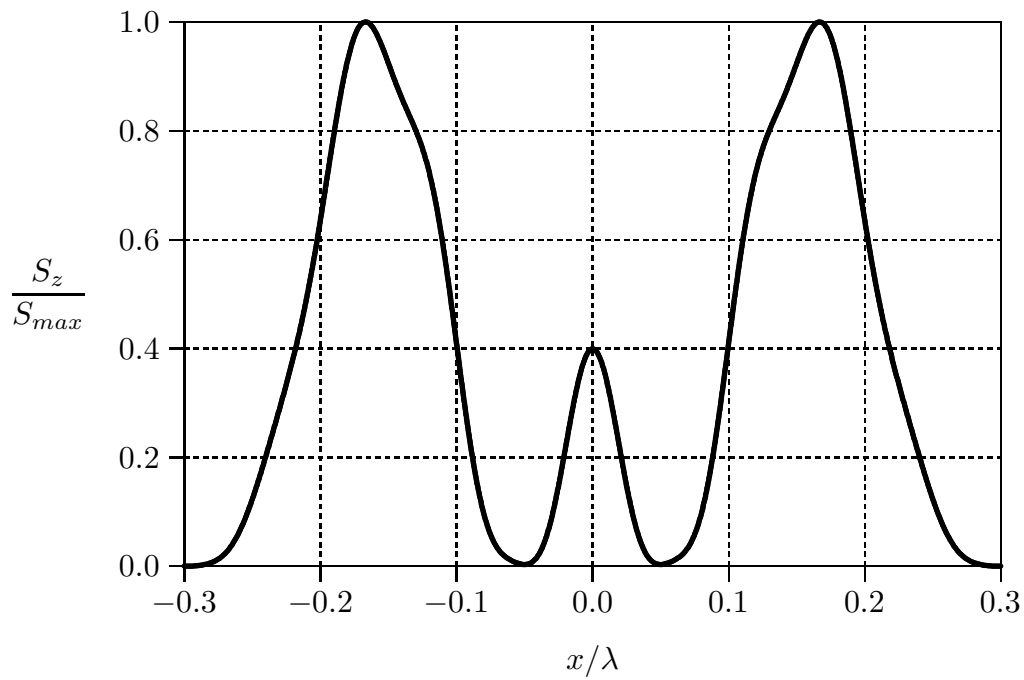
Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 9\lambda$, $l = 7\lambda$, $z = 1\lambda$



Diffrazione di Fresnel dell'horn circolare nel piano xy

$\nu = 10 \text{ GHz}$, $\lambda = 3 \text{ cm}$, $a = 10\lambda$, $l = 7\lambda$, $z = 1\lambda$



Diagrammi di radiazione 3D
Horn circolare: Diffrazione di Fresnel ($z=1\lambda$)

$\nu=10$ GHz, $\lambda=3$ cm, $a=0.5\lambda(1.5$ cm), $l=7\lambda(21$ cm)

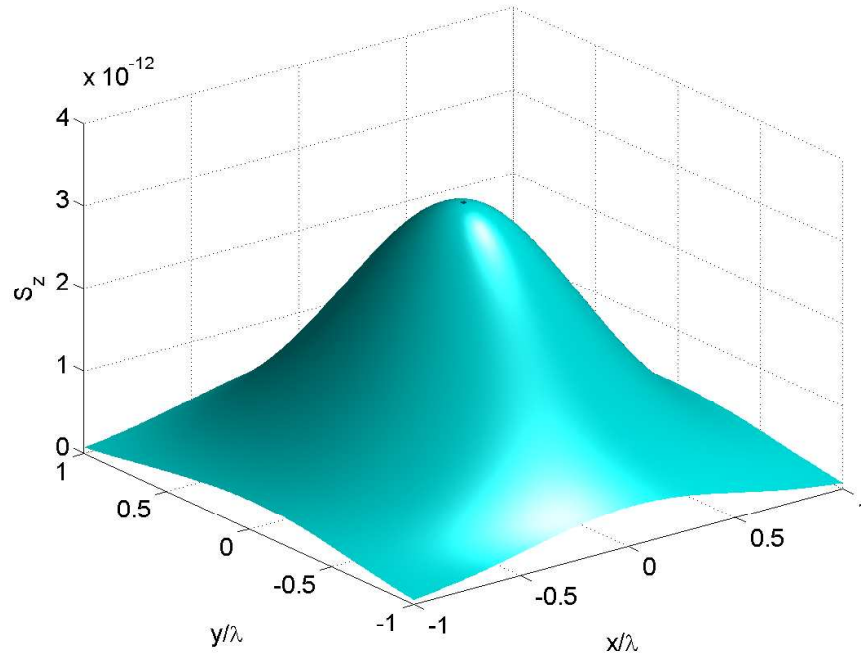


fig.12.6-1

$\nu=10$ GHz, $\lambda=3$ cm, $a=1\lambda(3$ cm), $l=7\lambda(21$ cm)

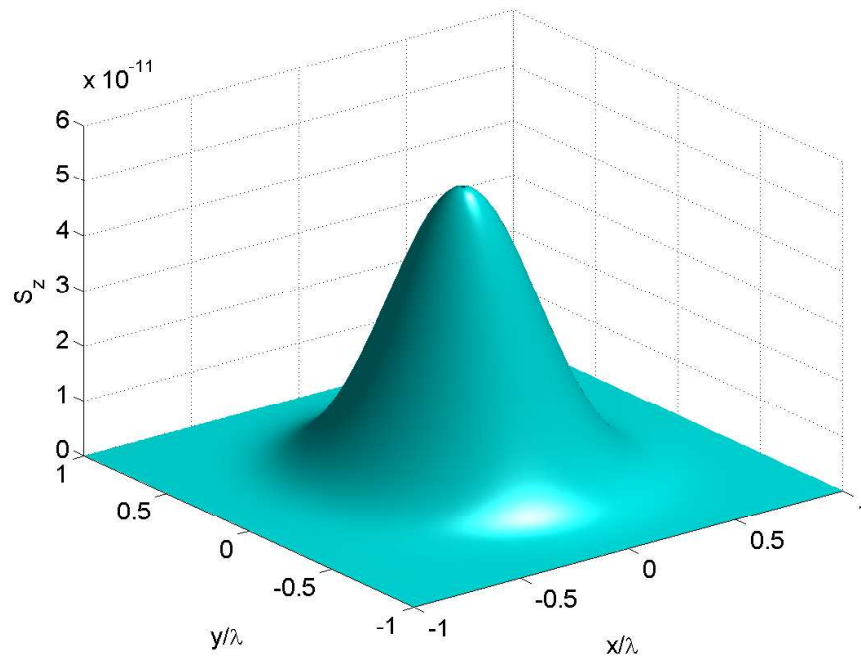


fig.12.6-2

$\nu=10$ GHz, $\lambda=3$ cm, $a=1.5\lambda(4.5$ cm), $l=7\lambda(21$ cm)

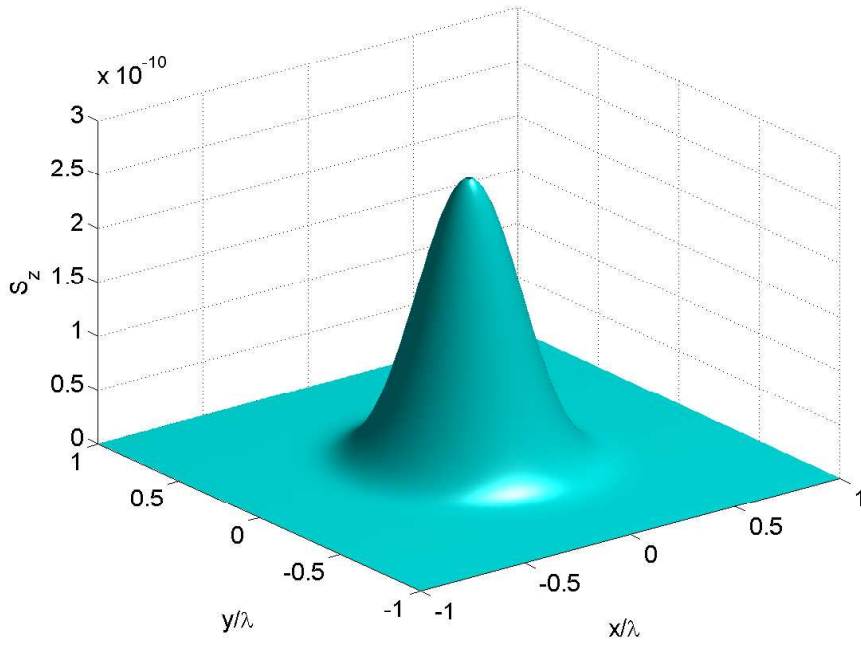


fig.12.6-3

$\nu=10$ GHz, $\lambda=3$ cm, $a=2\lambda(6$ cm), $l=7\lambda(21$ cm)

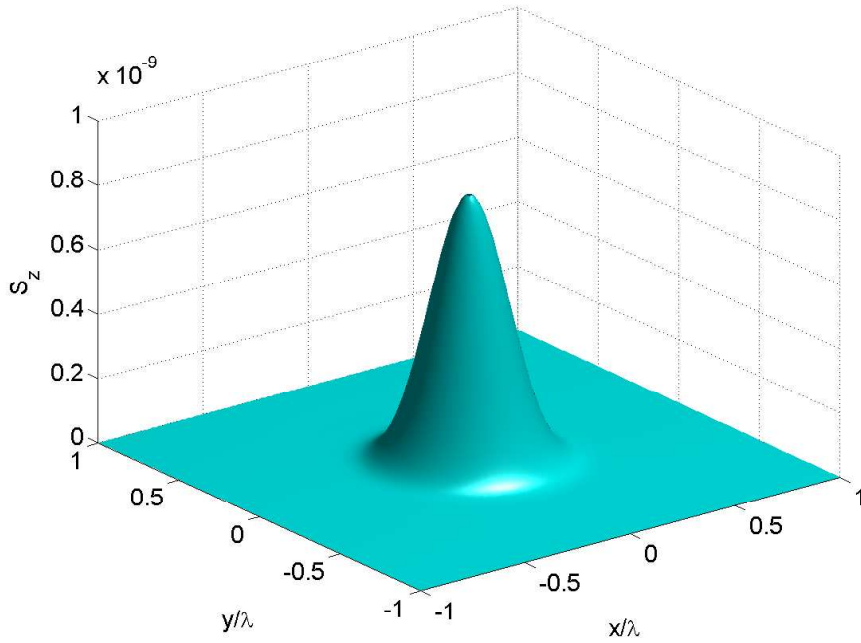


fig.12.6-4

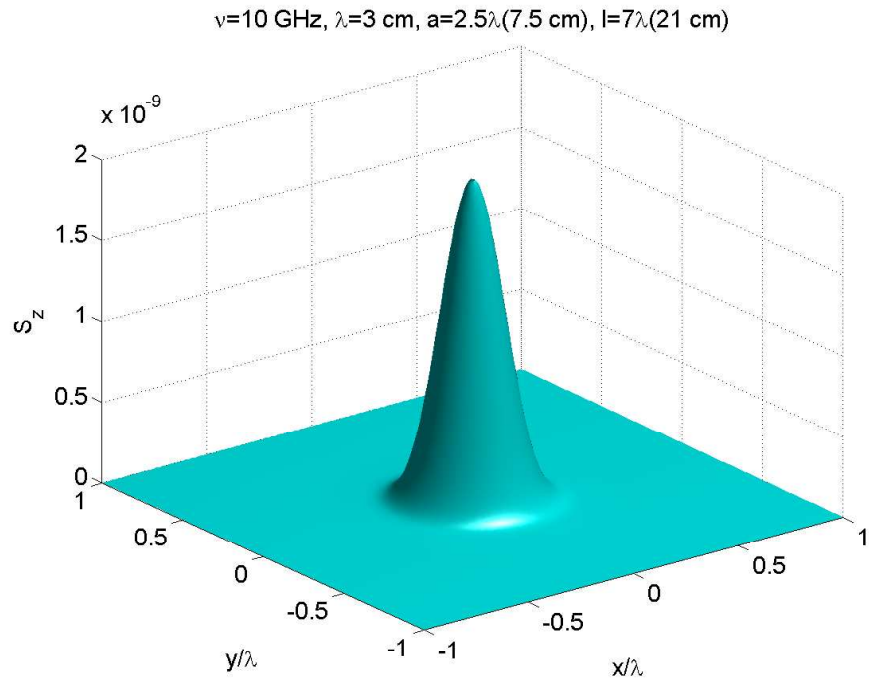


fig.12.6-5

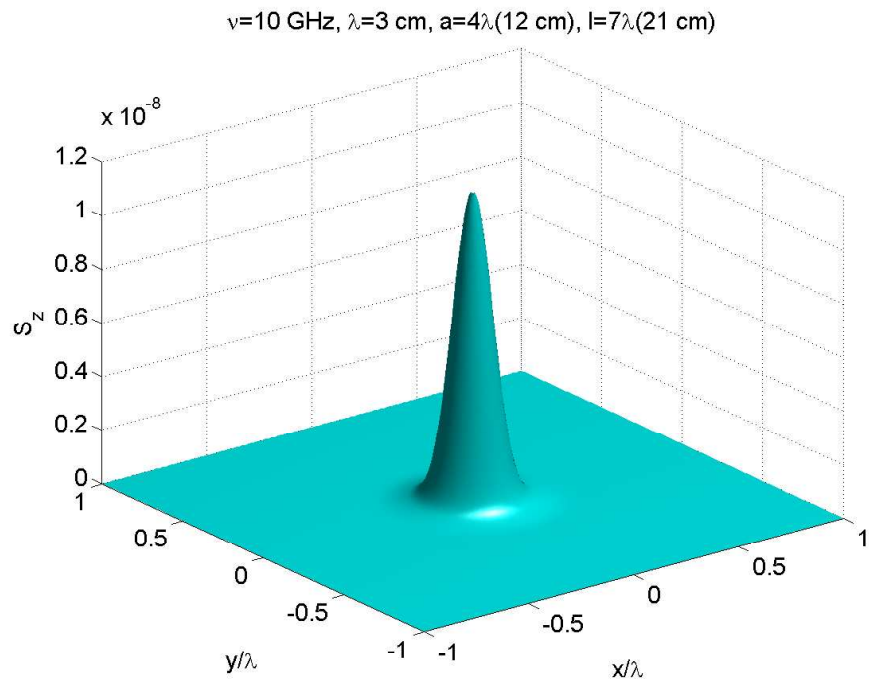


fig.12.6-6

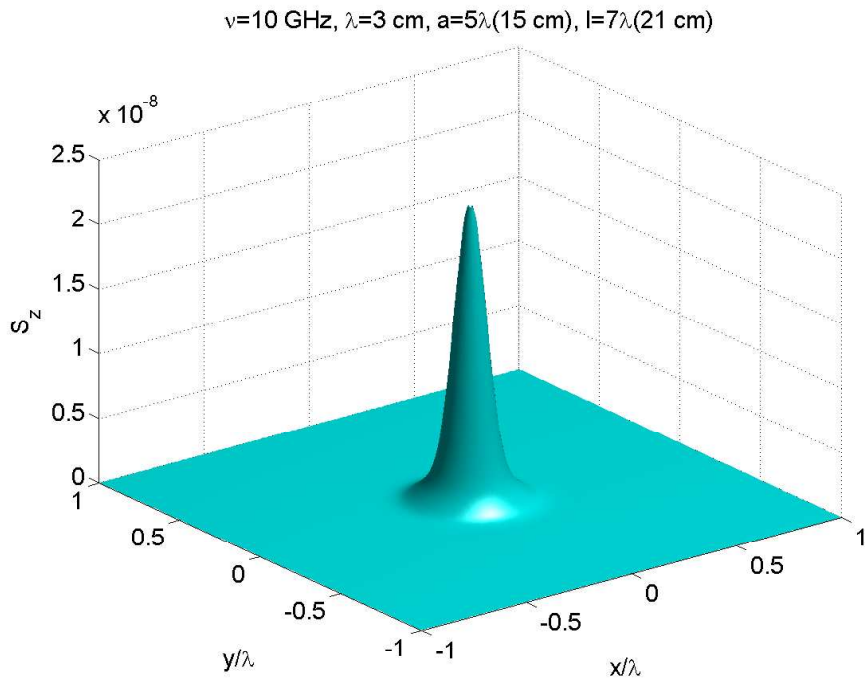


fig.12.6-7

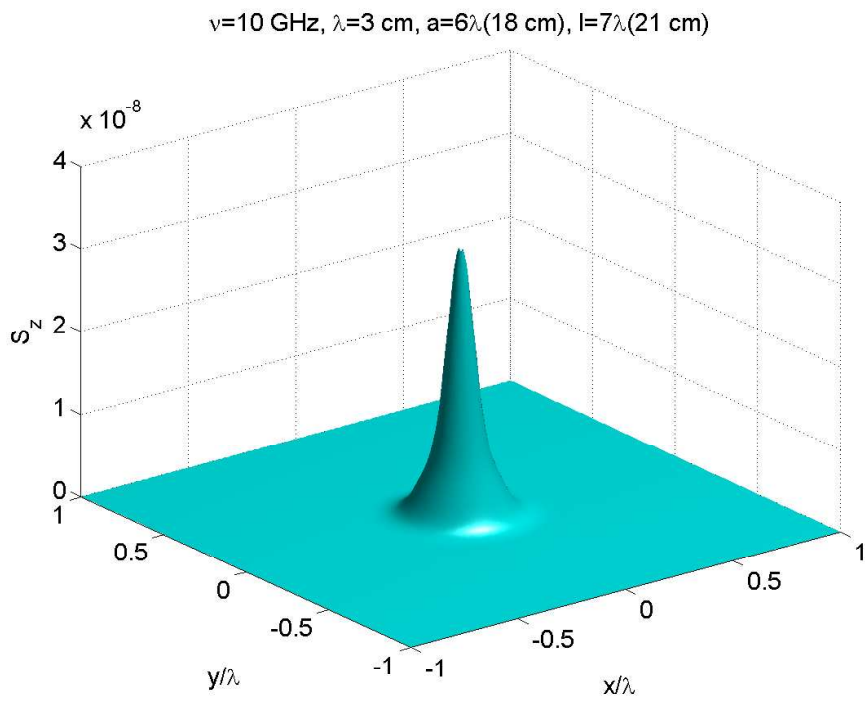


fig.12.6-8

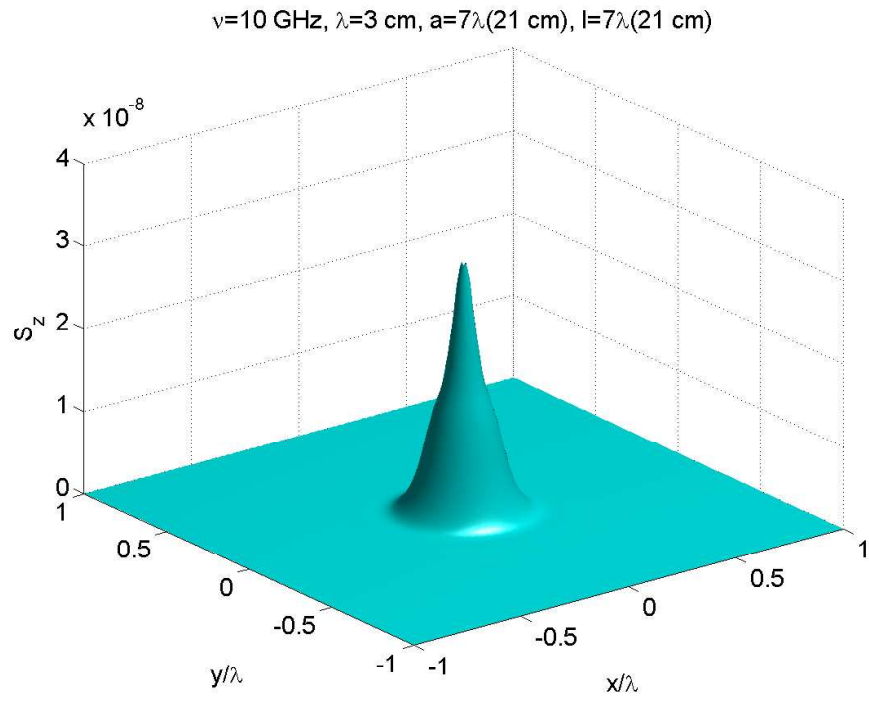


fig.12.6-9

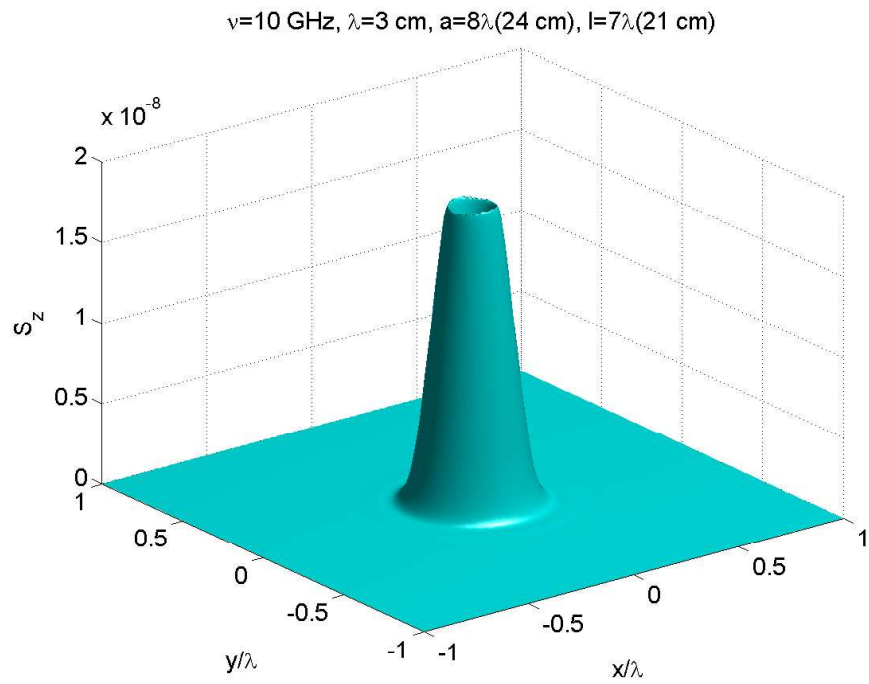


fig.12.6-10

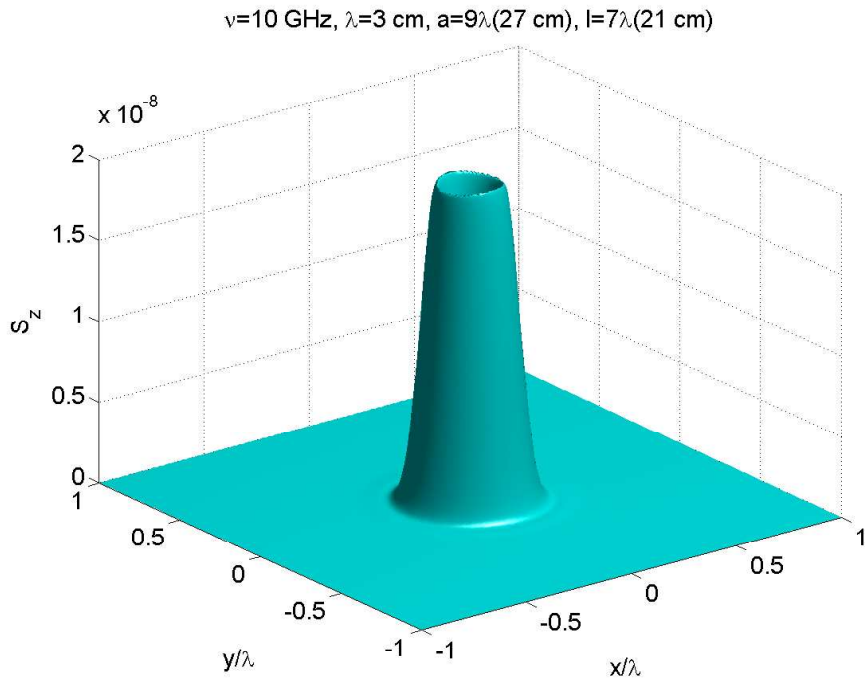


fig.12.6-11

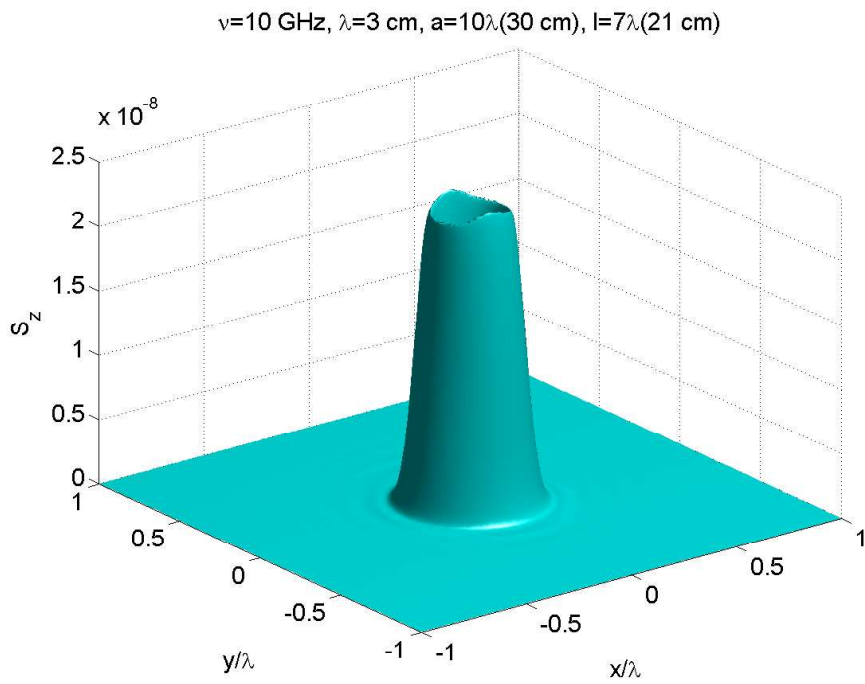


fig.12.6-12

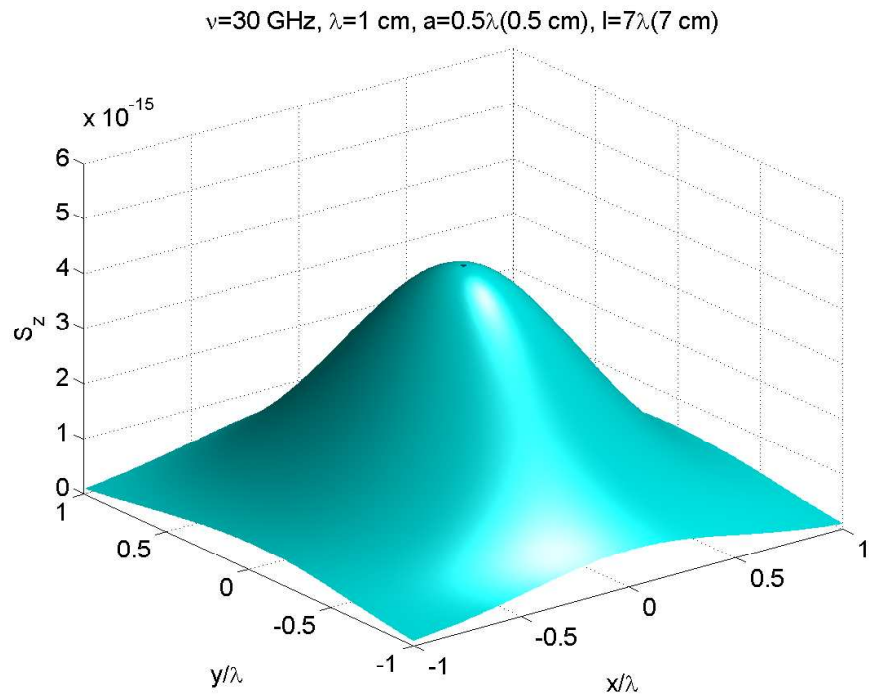


fig.12.6-13

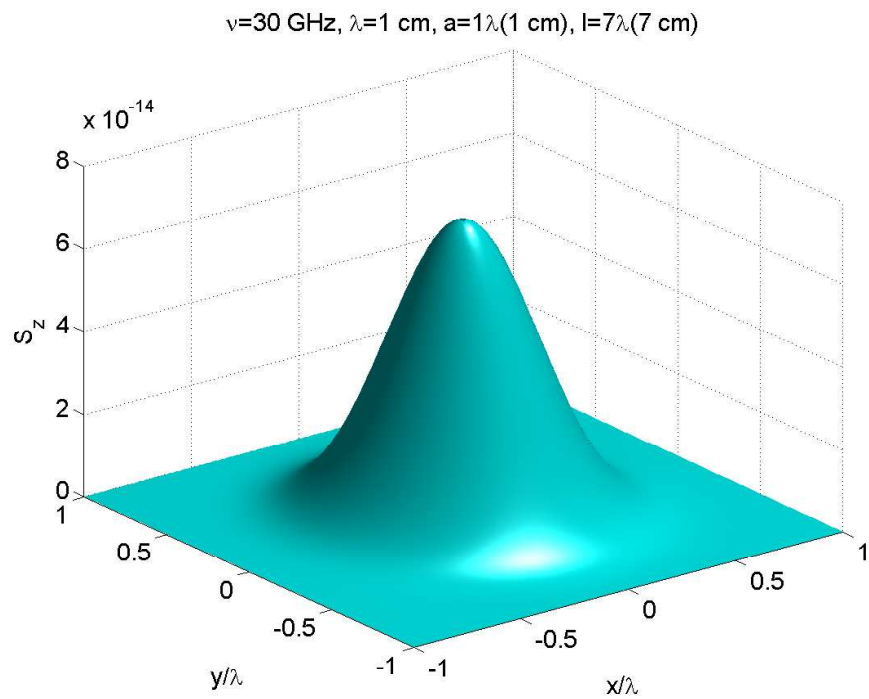


fig.12.6-14

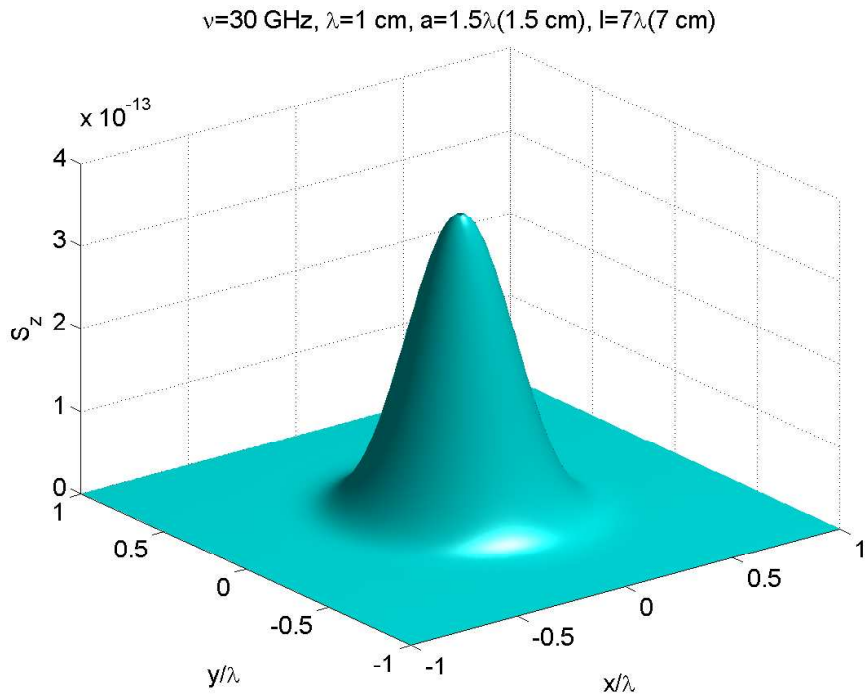


fig.12.6-15

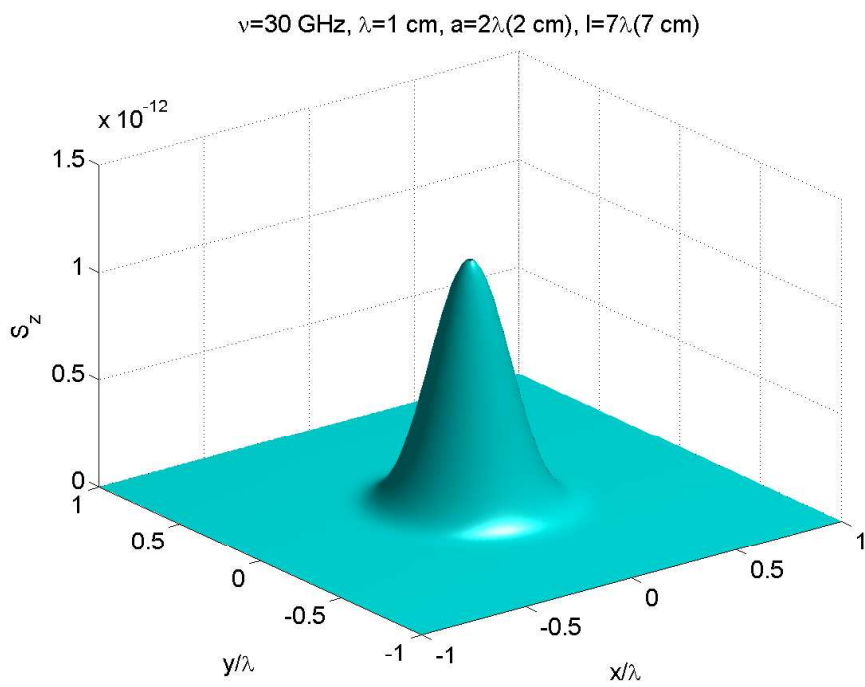


fig.12.6-16

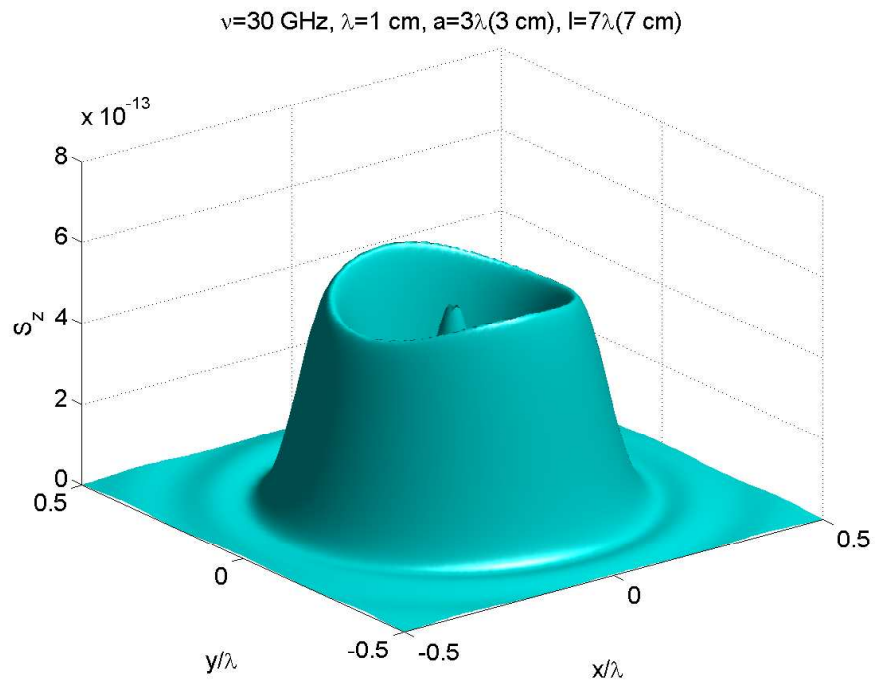


fig.12.6-17

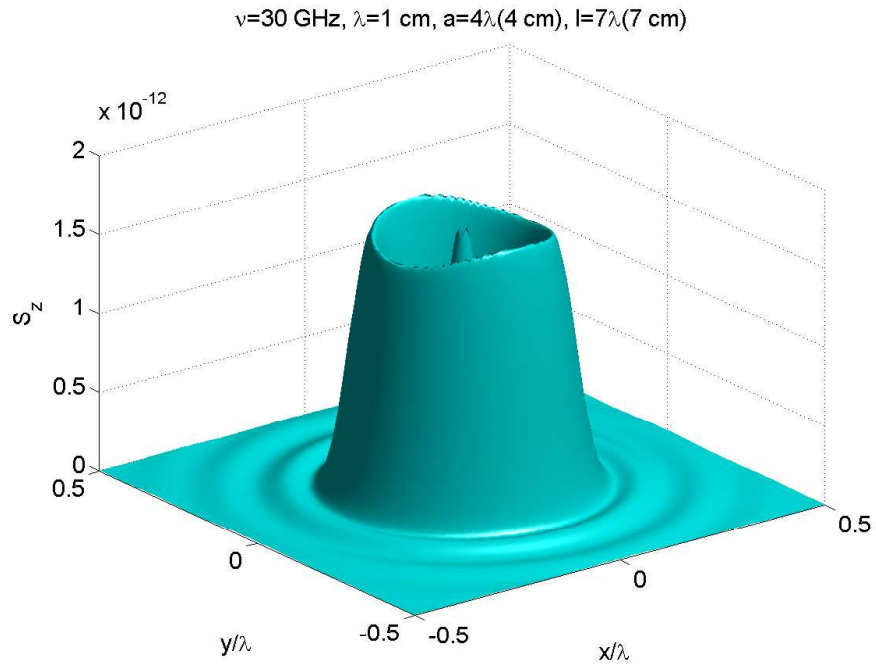


fig.12.6-18

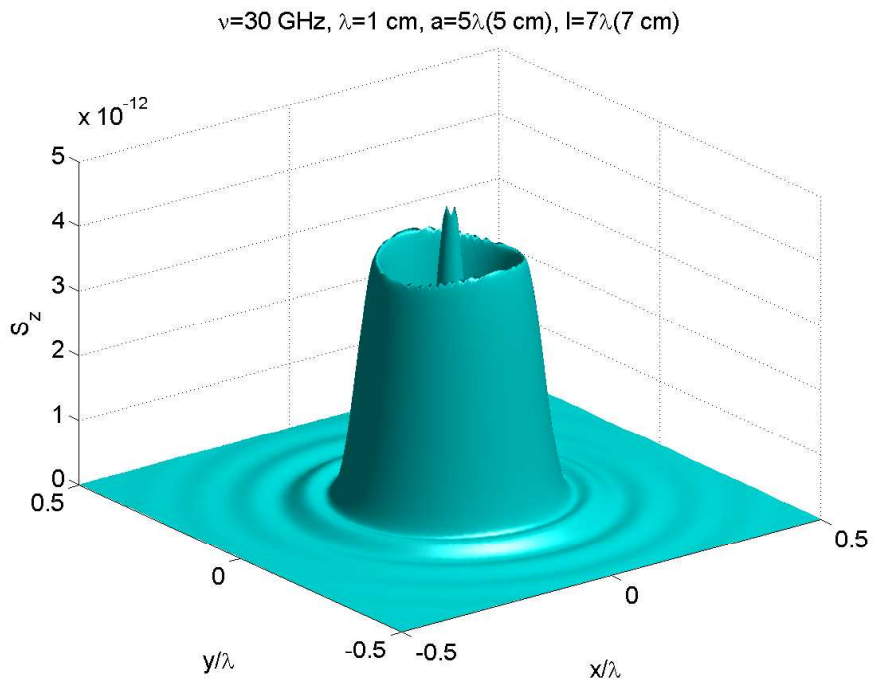


fig.12.6-19

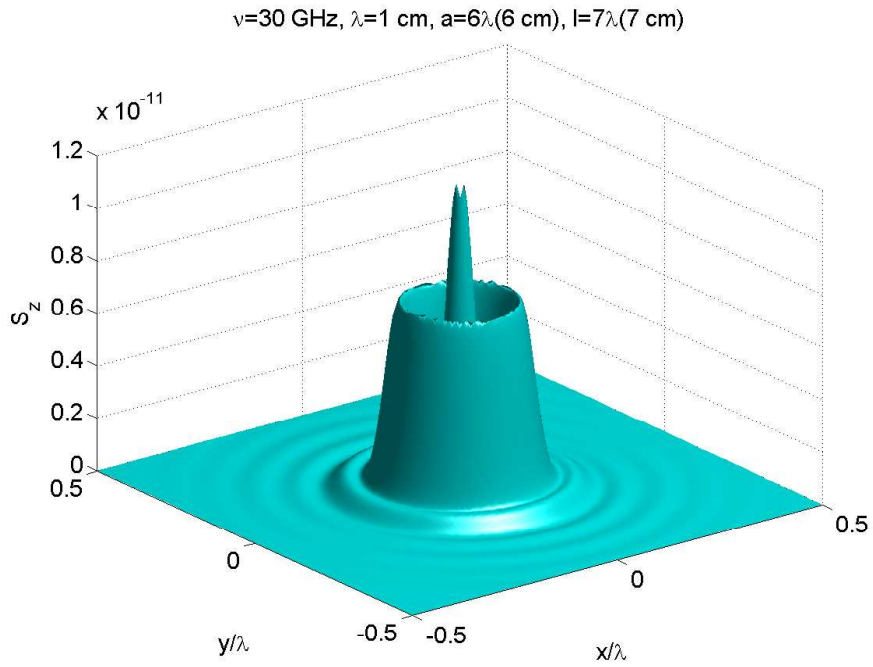


fig.12.6-20

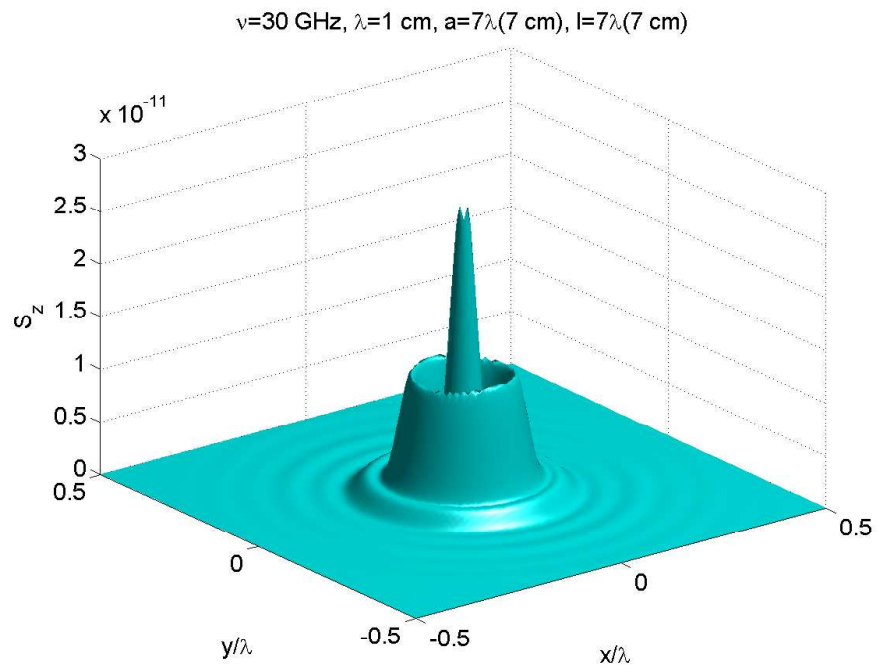


fig.12.6-21

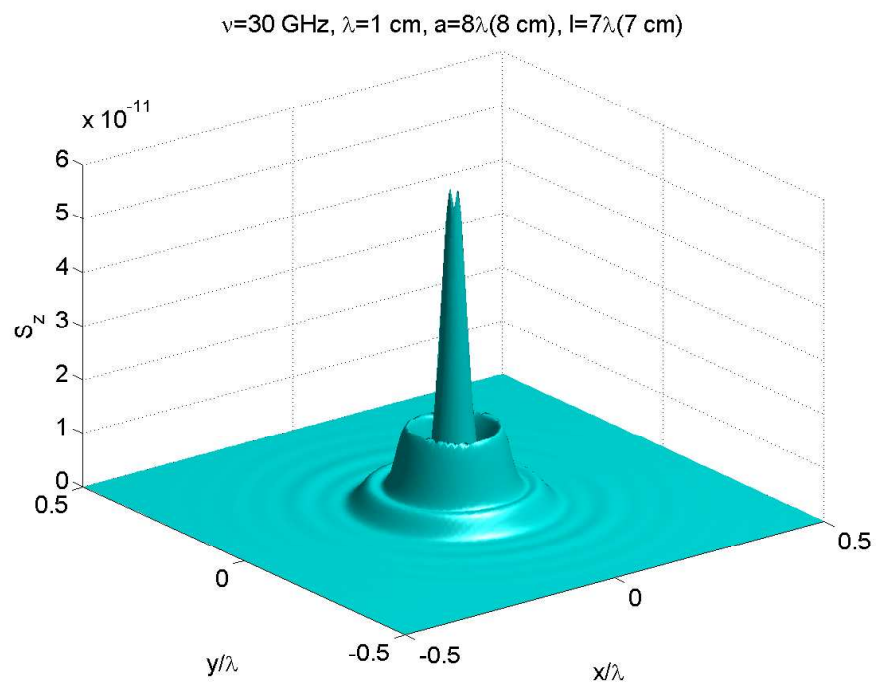


fig.12.6-22

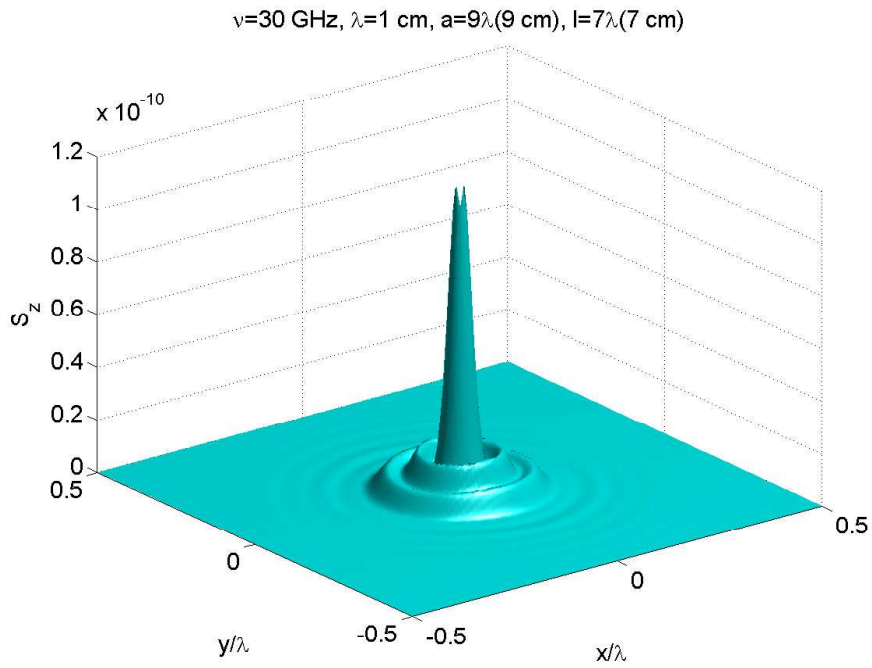


fig.12.6-23

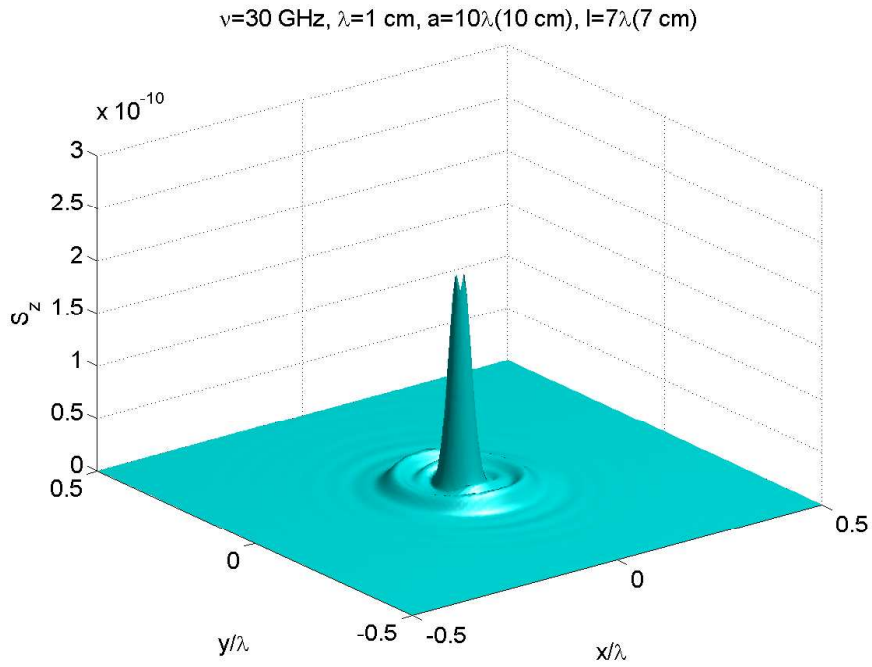


fig.12.6-24

Programmi per Diagrammi di Radiazione 3D in ambiente Matlab
Horn circolare: Diffrazione di Fresnel ($\nu = 10 \text{ GHz}$, $z = \lambda$)

$$a/\lambda = 0.5$$

```

01) -
02) - delete(get(0,'children'));
03) - clear all;
04) - c=3e8;
05) - freq=10e9;
05) - lambda=c./freq;
06) - z=1.*lambda; %%%%% distanza dall'horn %%%%%
07) - a=0.5.*lambda; %%%%% raggio dell'horn %%%%%
08) - l=7.*lambda; %%%%% lunghezza dell'horn per freq=10 GHz%%%%
09) - xp11=1.841;
10) - k=(2.*pi)./lambda;
11) - Ris1=0.01.*lambda;
12) - [X,Y] = meshgrid([-1.*lambda:Ris1:1.*lambda]);
13) - r=sqrt(X.^2+Y.^2+z.^2);
14) - ct=z./r; %%% costheta %%%
15) - s=sqrt(1-ct.^2); %%% sintheta %%%
16) - cf=X./r./s; %%% cosphi %%%
17) - sf=Y./r./s; %%% sinphi %%%
18) - sduef=2.*sf.*cf; %%% sin2phi %%%
19) - cduef=cf.^2-sf.^2; %%% cos2phi %%%
20) - %%% Integrazione %%%
21) - Ris=100; % precisione di integrazione
22) - wmax=1;
23) - for k1=1:Ris+1
24) - w(k1)=0+(k1-1).*((wmax-0)./Ris);
25) - end
26) - W0=0;
27) - W2=0;
28) - for j = 2:Ris+1
29) -
30) - W0=W0+exp(-i.*k.*r.^2.*s.^2./2./z).*pi.*
31) - (besselj(0,w(j).*xp11).*besselj(0,k.*a.*w(j).*r./z.*s)...
32) - .*exp(-i.*k.*a.^2.*w(j).*r./2.*(1./l+1./z)).*w(j).*
33) - a.^2.*(r(j)-r(j-1))+cduef.*...
34) - w(j).*a.^2.*besselj(2,w(j).*xp11).*besselj(2,k.*a.*r./z.*w(j).*s)...
35) - .*exp(-i.*k.*a.^2.*w(j).^2./2.*(1./l+1./z)).*(r(j)-r(j-1)));
36) -
37) - W2=W2+sduef.*exp(-i.*k.*r.^2.*s.^2./2./z).*
38) - w(j).*a.^2.*besselj(2,w(j).*xp11).*besselj(2,k.*a.*r./z.*w(j).*s)...

```

```
39) - .*exp(-i.*k.*a.^2.*w(j).^2./2.*(1./l+1./z)).*(r(j)-r(j-1));
40) - end
41) - Poynting=ct.^2.*X.^2./r.^2./s.^2.*
42) - (abs(W0+W2)).^2+Y.^2./r.^2./s.^2.*
43) - (abs(W0-W2)).^2;
44) - Poynting_z=Poynting.*ct;
45) - Poyntingmax=max(Poynting);
45) - surf(X./lambda,Y./lambda,Poynting_z,'FaceColor','cyan','EdgeColor','None')
46) - camlight right; lighting phong
47) - set(gca,'fontsize',15);
48) - xlabel('x/λ');
49) - ylabel('y/λ');
50) - zlabel('S_z');
51) - title('ν=10 GHz, λ=3 cm, a=0.5λ(1.5 cm), l=7λ(21 cm)')
52) - %view([60 90])
53) - print('-depsc2','horncircularfresnel3D2014_1new')
```

Fine del Cap.12